Statistical Process Control

SPC

Second Edition

STATISTICAL PROCESS CONTROL (SPC)

REFERENCE MANUAL

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STATISTICAL PROCESS CONTROL SPC

FOREWORD to Second Edition

This Reference Manual was developed by the Statistical Process Control (SPC) Work Group, sanctioned by the DaimlerChrysler/Ford/General Motors Supplier Quality Requirements Task Force, and under the auspices of the American Society for Quality (ASQ) and the Automotive Industry Action Group (AIAG). The Work Group responsible for this Second edition was prepared by the quality and supplier assessment staffs at DaimlerChrysler Corporation, Delphi Corporation, Ford Motor Company, General Motors Corporation, Omnex, Inc. and Robert Bosch Corporation working in collaboration with the Automotive Industry Action Group (AIAG).

The Task Force charter is to standardize the reference manuals, reporting formats and technical nomenclature used by DaimlerChrysler, Ford and General Motors in their respective supplier assessment systems. Accordingly, this Reference Manual can be used by any supplier to develop information responding to the requirements of either DaimlerChrysler's, Ford's or General Motors' supplier assessment systems. This second edition was prepared to recognize the needs and changes within the automotive industry in SPC techniques that have evolved since the original manual was published in 1991.

The manual is an introduction to statistical process control. ***It is not intended to limit evolution of SPC methods suited to particular processes or commodities.*** While these guidelines are intended to cover normally occurring SPC system situations, there will be questions that arise. These questions should be directed to your customer's Supplier Quality Assurance (SQA) activity. If you are uncertain as to how to contact the appropriate SQA activity, the buyer in your customer's purchasing office can help.

The Task Force gratefully acknowledges: the leadership and commitment of Vice Presidents Peter Rosenfeld at DaimlerChrysler Corporation, Thomas K. Brown at Ford Motor Company and Bo Andersson of General Motors Corporation; the assistance of the AIAG in the development, production and distribution of the manual; the guidance of the Task Force principals Hank Gryn (DaimlerChrysler Corporation), Russ Hopkins (Ford Motor Company), and Joe Bransky (General Motors Corporation). Therefore this manual was developed to meet the specific needs of the automotive industry.

ACKNOWLEDGEMENTS to Second Edition

The joint consensus on the contents of this document was effected through Task Team Subcommittee Members representing DaimlerChrysler, Ford, and General Motors, respectively, whose approval signatures appear below, and who gratefully acknowledge the significant contribution of Gregory Gruska of Omnex Inc., Gary A. Hiner of Delphi Corporation, and David W. Stamps of The Robert Bosch Corp.

The latest improvements were updating the format to conform to the current AIAG/ ISO/ TS 16949:2002 documentation, more clarification and examples to make the manual more user friendly and additional areas which where not included or did not exist when the original manual was written.

The current re-write subcommittee is chaired by Mike Down from General Motors Corporation and consists of Todd Kerkstra and Dave Benham from DaimlerChrysler Corporation, Peter Cvetkovski from Ford Motor Company, Gregory Gruska, as a representative of the Omnex Inc. and ASQ, Gary A. Hiner of Delphi Corporation, and David W. Stamps of The Robert Bosch Corp.

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STATISTICAL PROCESS CONTROL SPC

FOREWORD to First Edition

This Reference Manual was prepared by the quality and supplier assessment staffs at Chrysler, Ford and General Motors, working under the auspices of the Automotive Division of the American Society for Quality Control Supplier Quality Requirements Task Force, in collaboration with the Automotive Industry Action Group.

The ASQC/AIAG Task Force charter is to standardize the reference manuals, reporting formats and technical nomenclature used by Chrysler, Ford and General Motors in their respective supplier assessment systems: Supplier Quality Assurance, Total Quality Excellence and Targets for Excellence. Accordingly, this Reference Manual can be used by any supplier to develop information responding to the requirements of either Chrysler's, Ford's or General Motors' supplier assessment systems. Until now, there has been no unified formal approach in the automotive industry on statistical process control. Certain manufacturers provided methods for their suppliers, while others had no specific requirements. In an effort to simplify and minimize variation in supplier quality requirements, Chrysler, Ford, and General Motors agreed to develop and, through AIAG, distribute this manual. The work team responsible for the Manual's content was led by Leonard A. Brown of General Motors. The manual should be considered an introduction to statistical process control. It is not intended to limit evolution of statistical methods suited to particular processes or commodities nor is it intended to be comprehensive of all SPC techniques. Questions on the use of alternate methods should be referred to your customer's quality activity.

The Task Force gratefully acknowledges: the senior leadership and commitment of Vice Presidents Thomas T. Stallkamp at Chrysler, Clinton D. Lauer at Ford, and Donald A. Pais at General Motors; the technical competence and hard work of their quality and supplier assessment teams; and the invaluable contributions of the Automotive Industry Action Group (under AIAG Executive Director Joseph R. Phelan) in the development, production and distribution of this Reference manual. We also wish to thank the ASQC reading team led by Tripp Martin of Peterson Spring, who reviewed the Manual and in the process made valuable contributions to intent and content.

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December, 1991

ACKNOWLEDGEMENTS to First Edition

The joint consensus on the contents of this document was effected through Task Team Subcommittee Members representing General Motors, Ford, and Chrysler, respectively, whose approval signatures appear below, and who gratefully acknowledge the significant contribution of Pete Jessup of the Ford Motor Company, who was responsible for developing the majority of the material found in Chapters I, II, and III, and the Appendix of this document.

Harvey Goltzer of the Chrysler Corporation contributed concepts relative to process capability and capability studies, found in the introduction section of Chapter I. Jack Herman of Du Pont contributed some of the concepts relative to capability and performance indices and the importance of measurement variability, found in portions of Chapters II and IV, respectively.

The General Motors Powertrain Division contributed the discussion and examples relative to subgrouping and process over-adjustment. The section in Chapter II which provides understanding of process capability and related issues was developed by the General Motors Corporate Statistical Review Committee. This committee also contributed to the development of Chapter IV, Process Measurement Systems Analysis, as well as to some Appendix items.

Finally, valuable input to all sections of the manual was provided by ASQC representatives Gregory Gruska, Doug Berg, and Tripp Martin.

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Victor W. Lowe, Jr
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CHAPTER I

Continual Improvement
and
Statistical Process Control

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Introduction

To prosper in today's economic climate, we – automotive manufacturers, suppliers and dealer organizations – must be dedicated to continual improvement. We must constantly seek more efficient ways to produce products and services. These products and services must continue to improve in value. We must focus upon our customers, both internal and external, and make customer satisfaction a primary business goal.

To accomplish this, everyone in our organizations must be committed to improvement and to the use of effective methods. This manual describes several basic statistical methods that can be used to make our efforts at improvement more effective. Different levels of understanding are needed to perform different tasks. This manual is aimed at practitioners and managers beginning the application of statistical methods. It will also serve as a refresher on these basic methods for those who are now using more advanced techniques. Not all basic methods are included here. Coverage of other basic methods (such as check sheets, flowcharts, Pareto charts, cause and effect diagrams) and some advanced methods (such as other control charts, designed experiments, quality function deployment, etc.) is available in books and booklets such as those referenced in Appendix H.

The basic statistical methods addressed in this manual include those associated with statistical process control and process capability analysis.

Chapter I provides background for process control, explains several important concepts such as special and common causes of variation. It also introduces the control chart, which can be a very effective tool for analyzing and monitoring processes.

Chapter II describes the construction and use of control charts for both variables[1] data and attributes data.

Chapter III describes other types of control charts that can be used for specialized situations – probability based charts, short-run charts, charts for detecting small changes, non-normal, multivariate and other charts.

Chapter IV addresses process capability analysis.

The Appendices address sampling, over-adjustment, a process for selecting control charts, table of constants and formulae, the normal table, a glossary of terms and symbols, and references.

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1 The term "Variables", although awkward sounding, is used in order to distinguish the difference between something that varies, and the control chart used for data taken from a continuous variable.

Six Points

Six points should be made before the main discussion begins:

1) Gathering data and using statistical methods to interpret them are not ends in themselves. The overall aim should be increased understanding of the reader's processes. It is very easy to become technique experts without realizing any improvements. Increased knowledge should become a basis for action.

2) Measurement systems are critical to proper data analysis and they should be well understood before process data are collected. When such systems lack statistical control or their variation accounts for a substantial portion of the total variation in process data, inappropriate decisions may be made. For the purposes of this manual, it will be assumed that this system is under control and is not a significant contributor to total variation in the data. The reader is referred to the *Measurement Systems Analysis* (MSA) Manual available from AIAG for more information on this topic.

3) The basic concept of studying variation and using statistical signals to improve performance can be applied to any area. Such areas can be on the shop floor or in the office. Some examples are machines (performance characteristics), bookkeeping (error rates), gross sales, waste analysis (scrap rates), computer systems (performance characteristics) and materials management (transit times). This manual focuses upon shop floor applications. The reader is encouraged to consult the references in Appendix H for administrative and service applications.

4) SPC stands for Statistical Process Control. Historically, statistical methods have been routinely applied to parts, rather than processes. Application of statistical techniques to control output (such as parts) should be only the first step. Until the processes that generate the output become the focus of our efforts, the full power of these methods to improve quality, increase productivity and reduce cost may not be fully realized.

5) Although each point in the text is illustrated with a worked-out example, real understanding of the subject involves deeper contact with process control situations. The study of actual cases from the reader's own job location or from similar activities would be an important supplement to the text. There is no substitute for hands-on experience.

6) This manual should be considered a first step toward the use of statistical methods. It provides generally accepted approaches, which work in many instances. However, there exist exceptions where it is improper to blindly use these approaches. This manual does not replace the need for practitioners to increase their knowledge of statistical methods and theory. Readers are encouraged to pursue formal statistical education. Where the reader's processes and application of statistical methods have

advanced beyond the material covered here, the reader is also encouraged to consult with persons who have the proper knowledge and practice in statistical theory as to the appropriateness of other techniques. In any event, the procedures used must satisfy the customer's requirements.

THE NEED FOR PROCESS CONTROL

Detection – Tolerates Waste

Prevention – Avoids Waste

CHAPTER I – Section A

Prevention Versus Detection

In the past, Manufacturing often depended on Production to make the product and on Quality Control to inspect the final product and screen out items not meeting specifications. In administrative situations, work is often checked and rechecked in efforts to catch errors. Both cases involve a strategy of detection, which is wasteful, because it allows time and materials to be invested in products or services that are not always usable.

It is much more effective to avoid waste by not producing unusable output in the first place – **a strategy of prevention.**

A prevention strategy sounds sensible – even obvious – to most people. It is easily captured in such slogans as, "Do it right the first time". However, slogans are not enough. What is required is an understanding of the elements of a statistical process control system. The remaining seven subsections of this introduction cover these elements and can be viewed as answers to the following questions:

- What is meant by a process control system?
- How does variation affect process output?
- How can statistical techniques tell whether a problem is local in nature or involves broader systems?
- What is meant by a process being in statistical control?
  What is meant by a process being capable?
- What is a continual improvement cycle, and what part can process control play in it?
- What are control charts, and how are they used?
- What benefits can be expected from using control charts?

As this material is being studied, the reader may wish to refer to the Glossary in Appendix G for brief definitions of key terms and symbols.

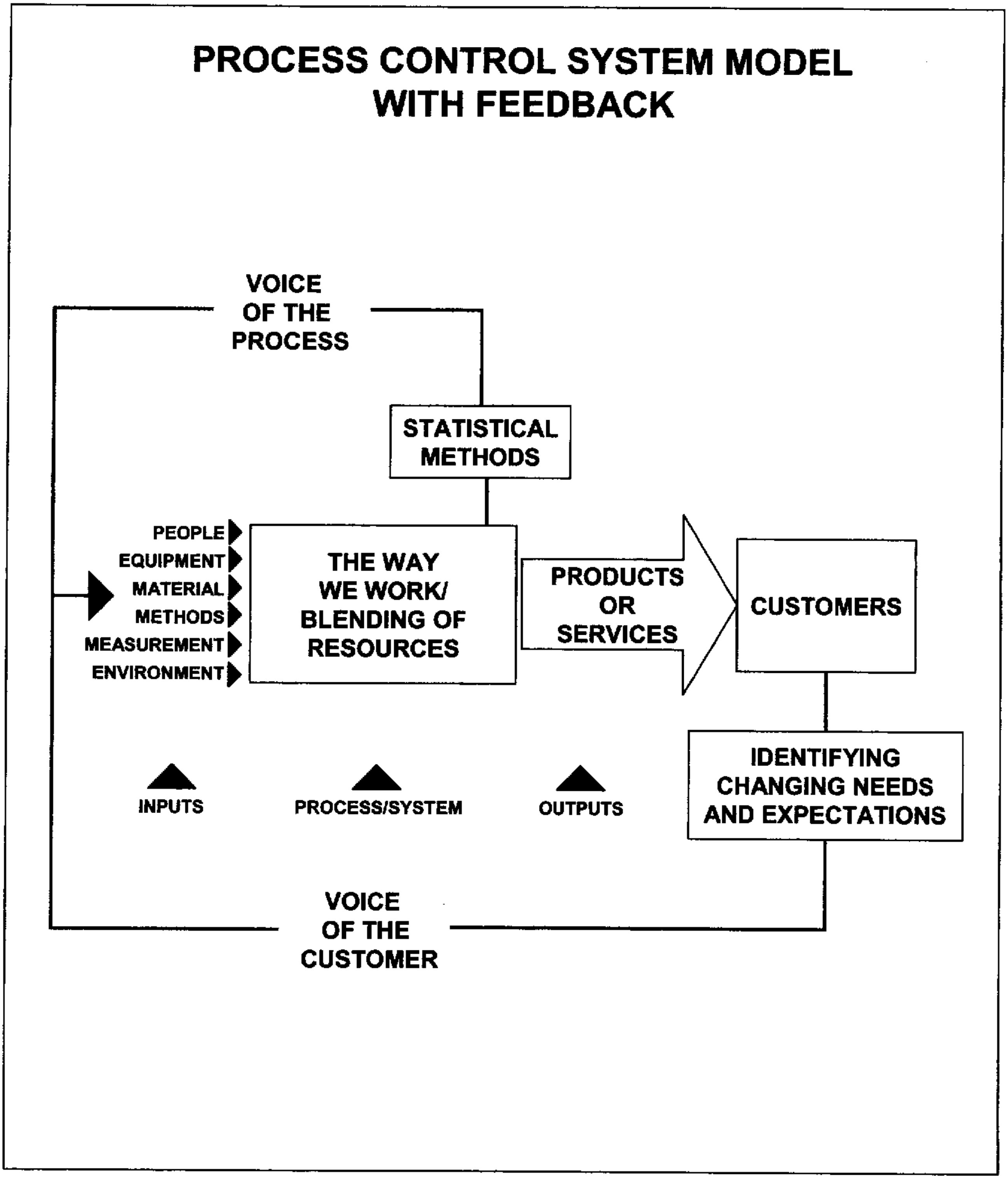

Figure I.1: A Process Control System

CHAPTER I – Section B

A Process Control System

A process control system can be described as a feedback system. SPC is one type of feedback system. Other such systems, which are not statistical, also exist. Four elements of that system are important to the discussions that will follow:

1. The Process – By the process, we mean the whole combination of suppliers, producers, people, equipment, input materials, methods, and environment that work together to produce output, and the customers who use that output (see Figure I.1). The total performance of the process depends upon communication between supplier and customer, the way the process is designed and implemented, and on the way it is operated and managed. The rest of the process control system is useful only if it contributes either to maintaining a level of excellence or to improving the total performance of the process.

2. Information About Performance – Much information about the actual performance of the process can be learned by studying the process output. The most helpful information about the performance of a process comes, however, from understanding the process itself and its internal variability. Process characteristics (such as temperatures, cycle times, feed rates, absenteeism, turnover, tardiness, or number of interruptions) should be the ultimate focus of our efforts. We need to determine the target values for those characteristics that result in the most productive operation of the process, and then monitor how near to or far from those target values we are. If this information is gathered and interpreted correctly, it can show whether the process is acting in a usual or unusual manner. Proper actions can then be taken, if needed, to correct the process or the just-produced output. When action is needed it must be timely and appropriate, or the information-gathering effort is wasted.

3. Action on the Process – Action on the process is frequently most economical when taken to prevent the important characteristics (process or output) from varying too far from their target values. This ensures the stability and the variation of the process output is maintained within acceptable limits. Such action might consist of:

- Changes in the operations
  - ✓ operator training
  - ✓ changes to the incoming materials
- Changes in the more basic elements of the process itself
  - ✓ the equipment
  - ✓ how people communicate and relate
  - ✓ the design of the process as a whole – which may be vulnerable to changes in shop temperature or humidity

The effect of actions should be monitored, with further analysis and action taken if necessary.

4. Action on the Output – Action on the output is frequently least economical when it is restricted to detecting and correcting out-of-specification product without addressing the underlying process problem. Unfortunately, if current output does not consistently meet customer requirements, it may be necessary to sort all products and to scrap or rework any nonconforming items. This must continue until the necessary corrective action on the process has been taken and verified.

It is obvious that inspection followed by action on only the output is a poor substitute for effective process management. Action on only the output should be used strictly as an interim measure for unstable or incapable processes (see Chapter I, Section E). Therefore, the discussions that follow focus on gathering process information and analyzing it so that action can be taken to correct the process itself. Remember, the focus should be on prevention not detection.

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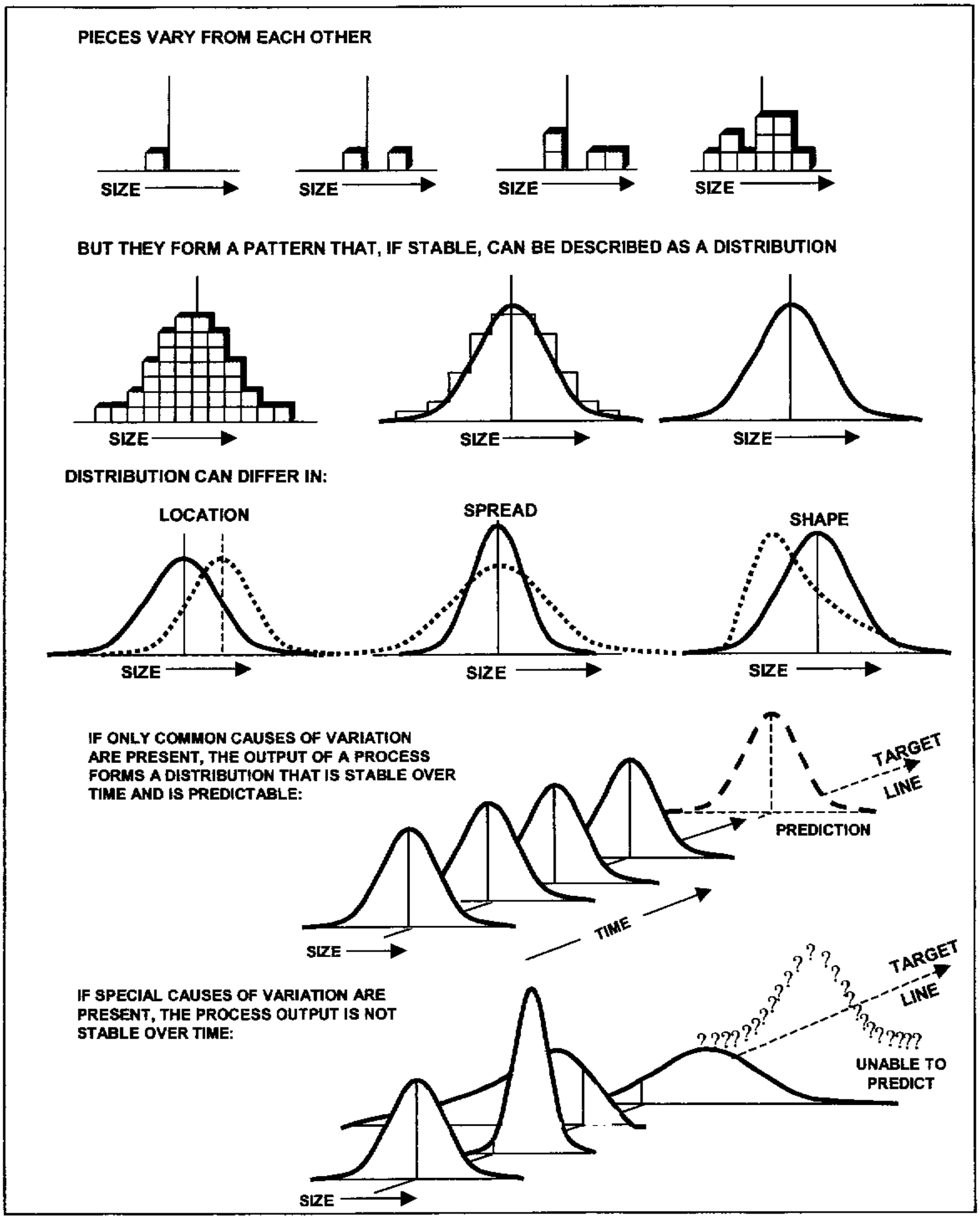

Figure I.2: Variation: Common Cause and Special Cause

CHAPTER I – Section C

Variation: Common and Special Causes

In order to effectively use process control measurement data, it is important to understand the concept of variation, as illustrated in Figure I.2.

No two products or characteristics are exactly alike, because any process contains many sources of variability. The differences among products may be large, or they may be immeasurably small, but they are always present. The diameter of a machined shaft, for instance, would be susceptible to potential variation from the machine (clearances, bearing wear), tool (strength, rate of wear), material (diameter, hardness), operator (part feed, accuracy of centering), maintenance (lubrication, replacement of worn parts), environment (temperature, constancy of power supply) and measurement system. Another example is the time required to process an invoice could vary according to the people performing various steps, the reliability of any equipment they were using, the accuracy and legibility of the invoice itself, the procedures followed, and the volume of other work in the office.

Some sources of variation in the process cause short-term, piece-to-piece differences, e.g., backlash and clearances within a machine and its fixturing, or the accuracy of a bookkeeper's work. Other sources of variation tend to cause changes in the output only over a longer period of time. These changes may occur either gradually as with tool or machine wear, stepwise as with procedural changes, or irregularly as with environmental changes such as power surges. Therefore, the time period and conditions over which measurements are made are critical since they will affect the amount of the total variation that will be observed.

While individual measured values may all be different, as a group they tend to form a pattern that can be described as a distribution (see Figure I.2). This distribution can be characterized by:

- Location (typical or "central" value)
- Spread (span or "width" of values from smallest to largest)
- Shape (the pattern of variation – whether it is symmetrical, skewed, etc.)

From the standpoint of minimum requirements, the issue of variation is often simplified: parts within specification tolerances are acceptable, parts beyond specification tolerances are not acceptable; reports on time are acceptable, late reports are not acceptable. *However, the goal should be to maintain the location to a target value with minimal variability.* To manage any process and reduce variation, the variation should be traced back to its sources. The first step is to make the distinction between common and special causes of variation.

Common causes refer to the many sources of variation that consistently acting on the process. Common causes within a process produce a stable and repeatable distribution over time. This is called "in a state of statistical control," "in statistical control," or sometimes just "in control." Common causes yield a *stable* system of chance causes. If only common causes of variation are present and do not change, the output of a process is predictable.

Special causes (often called assignable causes) refer to any factors causing variation that affect only some of the process output. They are often intermittent and unpredictable. Special causes are signaled by one or more points beyond the control limits or non-random patterns of points within the control limits. Unless all the special causes of variation are identified and acted upon, they may continue to affect the process output in unpredictable ways. If special causes of variation are present, the process output will not be stable over time.

The changes in the process distribution due to special causes can be either detrimental or beneficial. When detrimental, they need to be understood and removed. When beneficial, they should be understood and made a permanent part of the process. With some mature processes[2], the customer may give special allowance to run a process with a consistently occurring special cause. Such allowances will usually require that the process control plans can assure conformance to customer requirements and protect the process from other special causes (see Chapter I, Section E).

[2] Processes that have undergone several cycles of continual improvement.

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LOCAL ACTIONS AND ACTIONS ON THE SYSTEM

Local Actions

- Are usually required to eliminate special causes of variation
- Can usually be taken by people close to the process
- Can correct typically about 15% of process problems

Actions on the System

- Are usually required to reduce the variation due to common causes
- Almost always require management action for correction
- Are needed to correct typically about 85% of process problems

CHAPTER I – Section D

Local Actions And Actions On The System

There is an important connection between the two types of variation just discussed and the types of action necessary to reduce them.[3]

Simple statistical process control techniques can detect special causes of variation. Discovering a special cause of variation and taking the proper action is usually the responsibility of someone who is directly connected with the operation. Although management can sometimes be involved to correct the condition, the resolution of a special cause of variation usually requires local action, i.e., by people directly connected with the operation. This is especially true during the early process improvement efforts. As one succeeds in taking the proper action on special causes, those that remain will often require management action, rather than local action.

These same simple statistical techniques can also indicate the extent of common causes of variation, but the causes themselves need more detailed analysis to isolate. The correction of these common causes of variation is usually the responsibility of management. Sometimes people directly connected with the operation will be in a better position to identify them and pass them on to management for action. Overall, the resolution of common causes of variation usually requires action on the system.

Only a relatively small proportion of excessive process variation – industrial experience suggests about 15% – is correctable locally by people directly connected with the operation. The majority – the other 85% – is correctable only by management action on the system. Confusion about the type of action to take is very costly to the organization, in terms of wasted effort, delayed resolution of trouble, and aggravating problems. It may be wrong, for example, to take local action (e.g., adjusting a machine) when management action on the system is required (e.g., selecting suppliers that provide consistent input materials).[4] Nevertheless, close teamwork between management and those persons directly connected with the operation is a must for enhancing reduction of common causes of process variation.

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3 Dr. W. E. Deming has treated this issue in many articles; e.g., see Deming (1967).

4 These observations were first made by Dr. J. M. Juran, and have been borne out in Dr. Deming's experience.

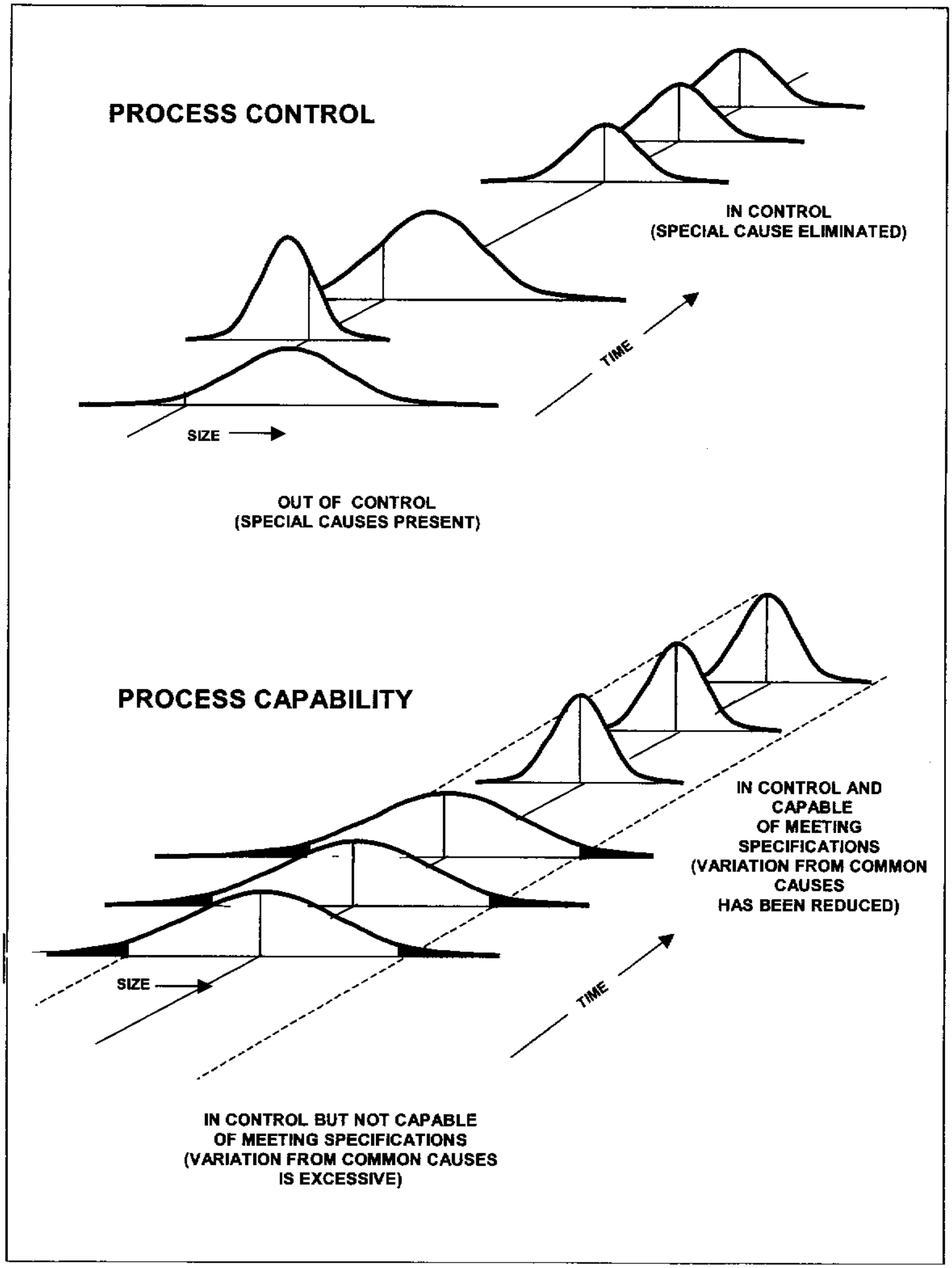

Figure I.3: Process Control and Process Capability

CHAPTER I – Section E

Process Control and Process Capability

The process control system is an integral part of the overall business management system.[5] As such, the goal of the process control system is to make predictions about the current and future state of the process. This leads to economically sound decisions about actions affecting the process. These decisions require balancing the risk of taking action when action is not necessary (over-control or "tampering") versus failing to take action when action is necessary (under-control).[6] These risks should be handled, however, in the context of the two sources of variation - special causes and common causes (see Figure I.3).

A process is said to be operating in statistical control when the only sources of variation are common causes. One function of a process control system, then, is to provide a statistical signal when special causes of variation are present, and to avoid giving false signals when they are not present. This allows appropriate action(s) to be taken upon those special causes (either removing them or, if they are beneficial, making them permanent).

The process control system can be used as a one-time evaluation tool but the real benefit of a process control system is realized when it is used as a continual learning tool instead of a conformance tool (good/bad, stable/not stable, capable/not capable, etc.)

Control vs. Capability

When discussing process capability, two somewhat contrasting concepts need to be considered:

- Process capability
- Process performance

Process capability is determined by the variation that comes from common causes. It generally represents the best performance of the process itself. This is demonstrated when the process is being operated in a state of statistical control regardless of the specifications.

Customers, internal or external, are however more typically concerned with the **process performance**; that is, the overall output of the process and how it relates to their requirements (defined by specifications), irrespective of the process variation.

[5] See TS 16949.
[6] See W. E. Deming, (1994), and W. Shewhart, (1931).

In general, since a process in statistical control can be described by a predictable distribution, the proportion of in-specification parts can be estimated from this distribution. As long as the process remains in statistical control and does not undergo a change in location, spread or shape, it will continue to produce the same distribution of in-specification parts.

Once the process is in statistical control the first action on the process should be to locate the process on the target. If the process spread is unacceptable, this strategy allows the minimum number of out-of-specification parts to be produced. Actions on the system to reduce the variation from common causes are usually required to improve the ability of the process (and its output) to meet specifications consistently. For a more detailed discussion of process capability, process performance and the associated assumptions, refer to Chapter IV.

The process must first be brought into statistical control by detecting and acting upon special causes of variation. Then its performance is predictable, and its capability to meet customer expectations can be assessed. This is a basis for continual improvement.

Every process is subject to classification based on capability and control. A process can be classified into 1 of 4 cases, as illustrated by the following chart:

Statistical Control

		In-Control	Out-of-Control
Capability	**Acceptable**	*Case 1*	*Case 3*
	Unacceptable	*Case 2*	*Case 4*

To be acceptable, the process must be in a state of statistical control and the capability (common cause variation) must be less than the tolerance. The ideal situation is to have a Case 1 process where the process is in statistical control and the ability to meet tolerance requirements is acceptable. A Case 2 process is in control but has excessive common cause variation, which must be reduced. A Case 3 process meets tolerance requirements but is not in statistical control; special causes of variation should be identified and acted upon. In Case 4, the process is not in control nor is it acceptable. Both common and special cause variation must be reduced.

Under certain circumstances, the customer may allow a producer to run a process even though it is a Case 3 process. These circumstances may include:

- The customer is insensitive to variation within specifications (see discussion on the loss function in Chapter IV).

- The economics involved in acting upon the special cause exceed the benefit to any and all customers. Economically allowable special causes may include tool wear, tool regrind, cyclical (seasonal) variation, etc.
- The special cause has been identified and has been documented as consistent and predictable.

In these situations, the customer may require the following:

- The process is mature.
- The special cause to be allowed has been shown to act in a consistent manner over a known period of time.
- A process control plan is in effect which will assure conformance to specification of all process output and protection from other special causes or inconsistency in the allowed special cause.

See also Appendix A for a discussion on time dependent processes.

Process Indices

The accepted practice in the automotive industry is to calculate the capability (common cause variation) only after a process has been demonstrated to be in a state of statistical control. These results are used as a basis for prediction of how the process will perform. There is little value in making predictions based on data collected from a process that is not stable and not repeatable over time. Special causes are responsible for changes in the shape, spread, or location of a process distribution, and thus can rapidly invalidate prediction about the process. **That is, in order for the various process indices and ratios to be used as *predictive tools*, the requirement is that the data used to calculate them are gathered from processes that are in a state of statistical control.**

Process indices can be divided into two categories: those that are calculated using within-subgroup estimates of variation and those using total variation when estimating a given index (see also chapter IV).

Several different indices have been developed because:

1) No single index can be universally applied to all processes, and
2) No given process can be completely described by a single index.

For example, it is recommended that C_p and C_{pk} both be used (see Chapter IV), and further that they be combined with graphical techniques to better understand the relationship between the estimated distribution and the specification limits. In one sense, this amounts to comparing (and trying to align) the "voice of the process" with the "voice of the customer" (see also Sherkenbach (1991)).

All indices have weaknesses and can be misleading. Any inferences drawn from computed indices should be driven by appropriate interpretation of the data from which the indices were computed.

Automotive companies have set requirements for process capability. It is the reader's responsibility to communicate with their customer and determine which indices to use. In some cases, it might be best to use no index at all. It is important to remember that most capability indices include the product specification in the formula. If the specification is inappropriate, or not based upon customer requirements, much time and effort may be wasted in trying to force the process to conform. Chapter IV deals with selected capability and performance indices and contains advice on the application of those indices.

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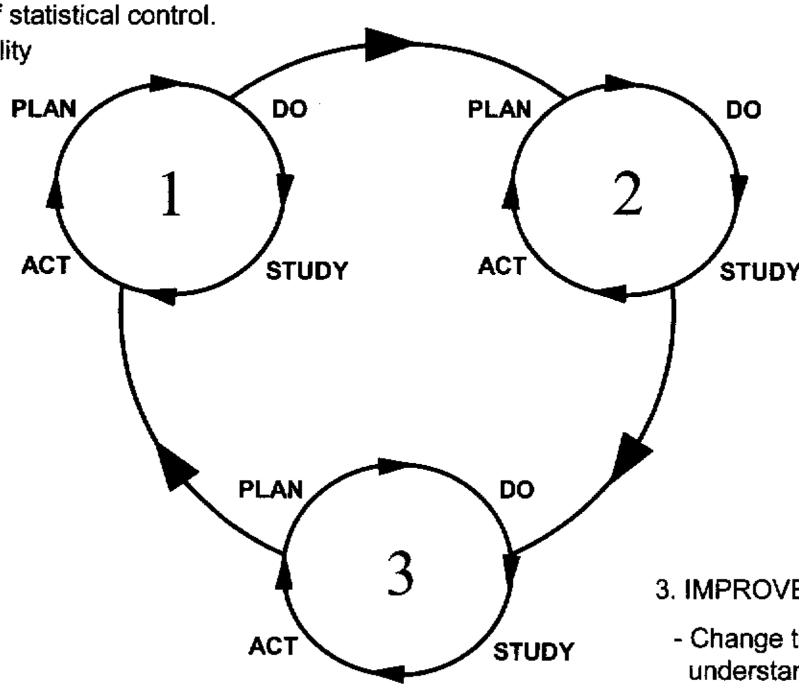
STAGES OF THE CONTINUAL PROCESS IMPROVEMENT CYCLE

1. ANALYZE THE PROCESS
   - What should the process be doing?
   - What can go wrong?
   - What is the process doing?
   - Achieve a state of statistical control.
   - Determine capability

2. MAINTAIN THE PROCESS
   - Monitor process performance
   - Detect special cause variation and act upon it.



3. IMPROVE THE PROCESS
   - Change the process to better understand common cause variation.
   - Reduce common cause variation.

Figure I.4: The Process Improvement Cycle

CHAPTER I – Section F

The Process Improvement Cycle and Process Control

In applying the concept of continual improvement to processes, there is a three-stage cycle that can be useful (see Figure I.4). Every process is in one of the three stages of the Improvement Cycle.

1. Analyze the Process

A basic understanding of the process is a must when considering process improvement. Among the questions to be answered in order to achieve a better understanding of the process are:

- What should the process be doing?
  - ✓ What is expected at each step of the process?
  - ✓ What are the operational definitions of the deliverables?
- What can go wrong?
  - ✓ What can vary in this process?
  - ✓ What do we already know about this process' variability?
  - ✓ What parameters are most sensitive to variation?
- What is the process doing?
  - ✓ Is this process producing scrap or output that requires rework?
  - ✓ Does this process produce output that is in a state of statistical control?
  - ✓ Is the process capable?
  - ✓ Is the process reliable?

Many techniques discussed in the *APQP Manual*[7] may be applied to gain a better understanding of the process. These activities include:

- Group meetings
- Consultation with people who develop or operate the process ("subject matter experts")
- Review of the process' history
- Construction of a Failure Modes and Effects Analysis (FMEA)

Control charts explained in this manual are powerful tools that should be used during the Process Improvement Cycle. These simple statistical methods help differentiate between common and special causes of variation. The special causes of variation must be addressed. When a state of statistical control has been reached, the process' current level of long-term capability can be assessed (see Chapter IV).

[7] Chrysler, Ford, and General Motors, (1995).

2. Maintain (Control) the Process

Once a better understanding of the process has been achieved, the process must be maintained at an appropriate level of capability. Processes are dynamic and will change. The performance of the process should be monitored so effective measures to prevent undesirable change can be taken. Desirable change also should be understood and institutionalized. Again, the simple statistical methods explained in this manual can assist. Construction and use of control charts and other tools will allow for efficient monitoring of the process. When the tool signals that the process has changed, quick and efficient measures can be taken to isolate the cause(s) and act upon them.

It is too easy to stop at this stage of the Process Improvement Cycle. It is important to realize that there is a limit to any company's resources. Some, perhaps many, processes should be at this stage. However, failure to proceed to the next stage in this cycle can result in a significant competitive disadvantage. The attainment of "world class" requires a steady and planned effort to move into the next stage of the Cycle.

3. Improve the Process

Up to this point, the effort has been to stabilize the processes and maintain them. However, for some processes, the customer will be sensitive even to variation within engineering specifications (see Chapter IV). In these instances, the value of continual improvement will not be realized until variation is reduced. At this point, additional process analysis tools, including more advanced statistical methods such as designed experiments and advanced control charts may be useful. Appendix H lists some helpful references for further study.

Process improvement through variation reduction typically involves purposefully introducing changes into the process and measuring the effects. The goal is a better understanding of the process, so that the common cause variation can be further reduced. The intent of this reduction is improved quality at lower cost.

When new process parameters have been determined, the Cycle shifts back to Analyze the Process. Since changes have been made, process stability will need to be reconfirmed. The process then continues to move around the Process Improvement Cycle.

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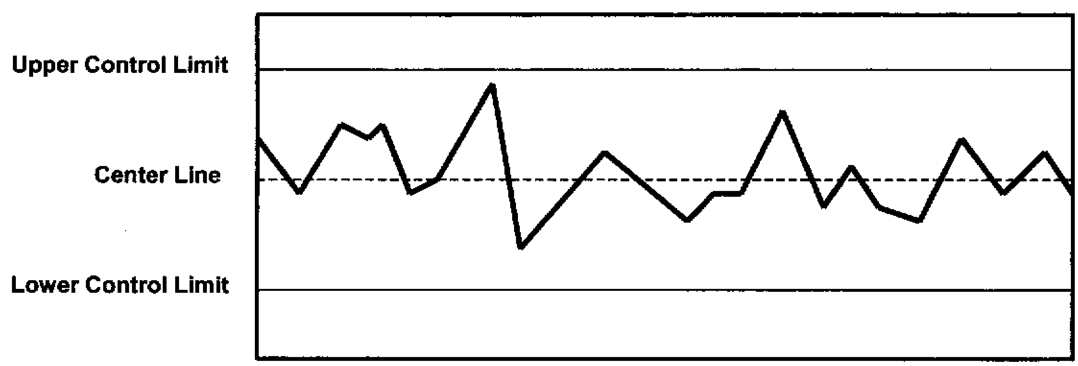
CONTROL CHARTS



1. **Collection**
    - Gather Data and plot on a chart.
2. **Control**
    - Calculate trial control limits from process data.
    - Identify special causes of variation and act upon them.
3. **Analysis and Improvement**
    - Quantify common cause variation; take action to reduce it.

These three phases are repeated for continual process improvement

Figure I.5: Control Charts

CHAPTER I – Section G

Control Charts:
Tools For Process Control and Improvement

In his books[8], Dr. W. E. Deming identifies two mistakes frequently made in process control:

> "Mistake 1. Ascribe a variation or a mistake to a special cause, when in fact the cause belongs to the system (common causes).
>
> Mistake 2. Ascribe a variation or a mistake to a system (common causes), when in fact the cause was special.
>
> Over adjustment [*tampering*] is a common example of mistake No. 1. Never doing anything to try to find a special cause is a common example of mistake No.2."

For effective variation management during production, there must be an effective means of detecting special causes. There is a common misconception that histograms can be used for this purpose. Histograms are the graphical representation of the distributional form of the process variation. The distributional form is studied to verify that the process variation is symmetric and unimodal and that it follows a normal distribution.

Unfortunately normality does not guarantee that there are no special causes acting on the process. That is, some special causes may change the process without destroying its symmetry or unimodality. Also a non-normal distribution may have no special causes acting upon it but its distributional form is non-symmetric.

Time-based statistical and probabilistic methods do provide necessary and sufficient methods of determining if special causes exist. Although several classes of methods are useful in this task, the most versatile and robust is the genre of control charts which were first developed and implemented by Dr. Walter Shewhart of the Bell Laboratories[9] while studying process data in the 1920's. He first made the distinction between controlled and uncontrolled variation due to what is called common and special causes. He developed a simple but powerful tool to separate the two – the control chart. Since that time, control charts have been used successfully in a wide variety of process control and improvement situations. Experience has shown that control charts effectively direct attention toward special causes of variation when they occur and reflect the extent of common cause variation that must be reduced by system or process improvement.

It is impossible to reduce the above mistakes to zero. Dr. Shewhart realized this and developed a graphical approach to minimize, over the long run, the economic loss from both mistakes.

---

[8] Deming (1989) and Deming (1994).
[9] Shewhart (1931).

If process control activities assure that no special cause sources of variation are active[10], the process is said to be in statistical control or "in control." Such processes are said to be stable, predictable, and consistent since it is possible to predict[11] the performance of the process.

The active existence of any special cause will render the process out of statistical control or "out of control." The performance of such unstable processes cannot be predicted.

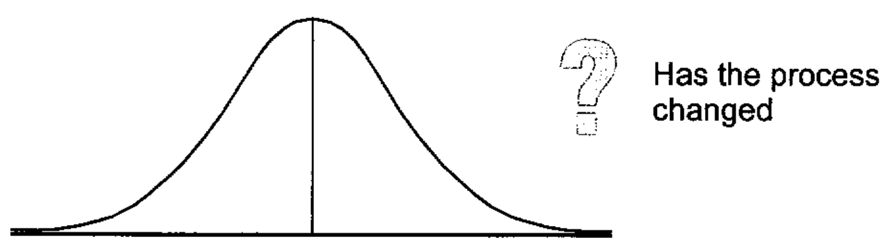
How do they work?

Control Limits

When Shewhart developed control charts he was concerned with the economic control of processes; i.e., action is taken on the process only when special causes are present. To do this, sample statistics are compared to control limits. But how are these limits determined?

Consider a process distribution that can be described by the normal form. The goal is to determine when special causes are affecting it. Another way of saying this is, "Has the process changed since it was last looked at it or during the period sampled?"

Shewhart's Two Rules for the Presentation of Data:
Data should always be presented in such a way that preserves the evidence in the data for all the predictions that might be made from these data.
Whenever an average, range, or histogram is used to summarize data, the summary should not mislead the user into taking any action that the user would not take if the data were presented in a time series.



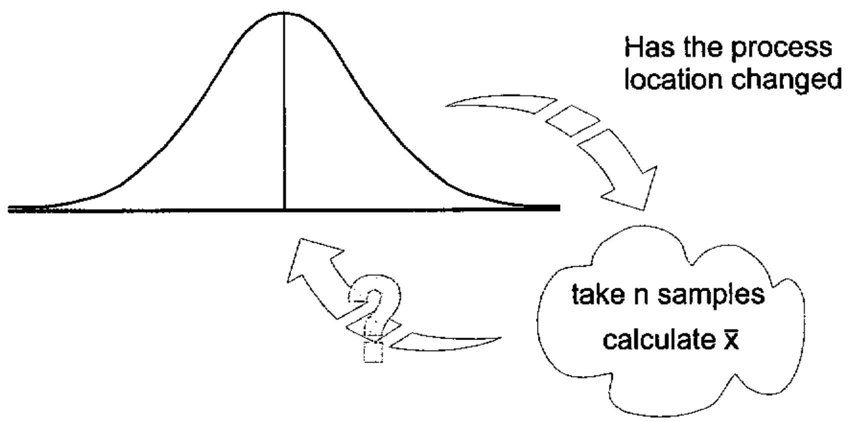
Since the normal distribution is described by its process location (mean) and process width (range or standard deviation) this question becomes: Has the process location or process width changed?

Consider only the location. What approach can be used to determine if the process location has changed? One possibility would be to look at

10 This is done by using the process information to identify and eliminate the existence of special causes or detecting them and removing their effect when they do occur.

11 As with all probabilistic methods some risk is involved. The exact level of belief in prediction of future actions cannot be determined by statistical measures alone. Subject-matter expertise is required.

every part produced by the process, but that is usually not economical. The alternative is to use a sample of the process, and calculate the mean of the sample.



If the process has ***not*** changed, will the sample mean be equal to the distribution mean?

The answer is that this very rarely happens. But how is this possible? After all, the process has not changed. Doesn't that imply that the process mean remains the same? The reason for this is that the sample mean is only an estimation of the process mean.

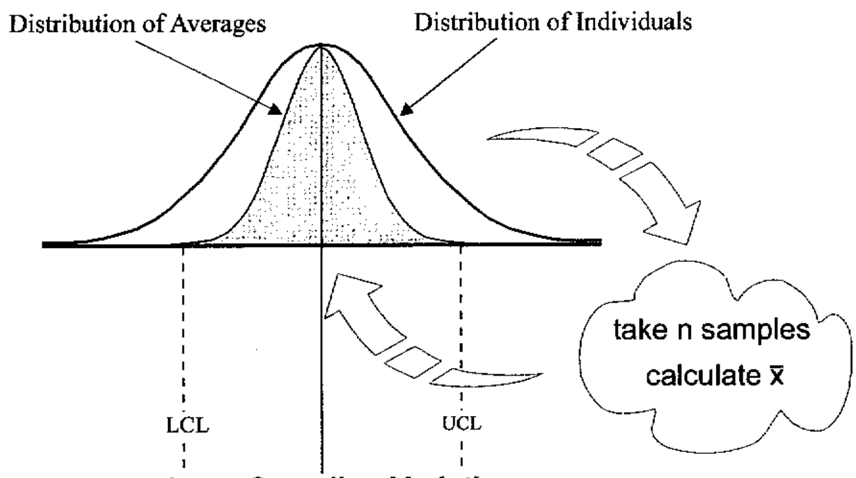
To make this a little clearer, consider taking a sample of size one. The mean of the sample is the individual sample itself. With such random samples from the distribution, the readings will eventually cover the entire process range. Using the formula:

$$\text{Range of the distribution of means} = \left(1/\sqrt{n}\right)\text{Process Range}$$

for a sample of size four, the resulting range of sample averages will be $1/\sqrt{4} = 1/2$ of the process range; for a sample of size 100 it will be $1/\sqrt{100} = 1/10$ of the process range.[12]

Shewhart used this sampling distribution to establish an operational definition of "in statistical control." First, start off with the assumption that the process is in statistical control, i.e., innocent until proven guilty. Then, compare the sample to the sampling distribution using the ±3 standard deviation limits[13]. These are called control limits. If the sample falls outside these limits then there is reason to believe that a special cause is present. Further, it is expected that all the (random) samples will exhibit a random ordering within these limits. If a group of samples shows a pattern there is reason to believe that a special cause is present. (see Chapter I, Section C, and Chapter II, Section A).

[12] See the Central Limit Theorem.
[13] Shewhart selected the ±3 standard deviation limits as useful limits in achieving the economic control of processes.



*due to **Sampling Variation***

In general, to set up a control chart we calculate:

Centerline = average of the statistic being analyzed

UCL = upper control limit = centerline + 3 x standard deviation of the averages

LCL = lower control limit = centerline - 3 x standard deviation of the averages

Approach:

Since Control Charts provide the operational definition of "in statistical control," they are useful tools at every stage of the Improvement Cycle (see Chapter I, Section F). Within each stage, the PDSA[14] cycle should be used.

For analysis of existing data sets

For the Analysis and Improvement stages of the cycle:

- Review the data:
  - ✓ Is the metric appropriate; i.e., does it reflect a process attribute and tied to a key business factor?
  - ✓ Are the data consistent; i.e., is the same operational definition used by all parties collecting the data?
  - ✓ Are the data reliable; i.e., is a planned data collection scheme utilized?
  - ✓ Is the measurement system appropriate and acceptable?
- Plot the data:
  - ✓ Plot using the time order
  - ✓ Compare to control limits and determine if there are any points outside the control limits

[14] Plan-Do-Study-Act cycle; also known as the PDCA, (Plan-Do-Check-Act) cycle.

  - ✓ Compare to the centerline and determine if there are any non-random patterns clearly discernible

- Analyze the data
- Take appropriate action

The data are compared with the control limits to see whether the variation is stable and appears to come from only common causes. If special causes of variation are evident, the process is studied to further determine what is affecting it. After actions (see Chapter I, Section D) have been taken, further data are collected, control limits are recalculated if necessary, and any additional special causes are acted upon.

After all special causes have been addressed and the process is running in statistical control, the control chart continues as a monitoring tool. Process capability can also be calculated. If the variation from common causes is excessive, the process cannot produce output that consistently meets customer requirements. The process itself must be investigated, and, typically, management action must be taken to improve the system.

For control

- Review the data collection scheme before starting:
  - ✓ Is the metric appropriate; i.e., does it reflect a process attribute and tied to a key business factor?
  - ✓ Will the data be consistent; i.e., is the same operational definition used by all parties collecting the data?
  - ✓ Will the data be reliable; i.e., is a planned data collection scheme used?
  - ✓ Is the measurement system appropriate and acceptable?
- Plot each point as it is determined:
  - ✓ Compare to control limits and determine if there are any points outside the control limits
  - ✓ Compare to the centerline and determine if there are any non-random patterns clearly discernible
- Analyze the data
- Take appropriate action:
  - ✓ Continue to run with no action taken; or
  - ✓ Identify source of the special cause and remove (if unacceptable response) or reinforce (if acceptable response); or
  - ✓ Continue to run with no action taken and reduce sample size or frequency; or
  - ✓ Initiate a continual improvement action

Often it is found that although the process was aimed at the target value during initial setup, the actual process location (μ)[15] may not

[15] The Greek letter μ is used to indicate the actual process mean, which is estimated by the sample mean $\bar{X}$.

match this value. For those processes where the actual location deviates from the target and the ability to relocate the process is economical, consideration should be given to adjusting the process so that it is aligned with the target (see Chapter IV, Section C). This assumes that this adjustment does not affect the process variation. This may not always hold true, but the causes for any possible increase in process variation after re-targeting the process should be understood and assessed against both customer satisfaction and economics.

The long-term performance of the process should continue to be analyzed. This can be accomplished by a periodic and systematic review of the ongoing control charts. New evidence of special causes might be revealed. Some special causes, when understood, will be beneficial and useful for process improvement Others will be detrimental, and will need to be corrected or removed.

The purpose of the Improvement Cycle is to gain an understanding of the process and its variability to improve its performance. As this understanding matures, the need for continual monitoring of ***product variables*** may become less – especially in processes where documented analysis shows that the dominant source of variation are more efficiently and effectively controlled by other approaches. For example: in processes where maintenance is the dominant source of variation, the process is best controlled by preventive and predictive maintenance; for processes where process setup is the dominant source of variation, the process is best controlled by setup control charts.

For a process that is in statistical control, improvement efforts will often focus on reducing the common cause variation in the process. Reducing this variation will have the effect of "shrinking" the control limits on the control chart (i.e., the limits, upon their recalculation, will be closer together). Many people, not familiar with control charts, feel this is "penalizing" the process for improving. They do not realize that if a process is stable and the control limits are calculated correctly, the chance that the process will erroneously yield an out-of-control point is the same regardless of the distance between the control limits (see Chapter I, Section E).

One area deserving mention is the question of recalculation of control chart limits. Once properly computed, and if no changes to the common cause variation of the process occur, then the control limits remain legitimate. Signals of special causes of variation do not require the recalculation of control limits. For long-term analysis of control charts, it is best to recalculate control limits as infrequently as possible; only as dictated by changes in the process.

For continual process improvement, repeat the three stages of the Improvement Cycle: Analyze the Process; Maintain (Control) the Process; Improve the Process, see Figure I.4.

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BENEFITS OF CONTROL CHARTS

Properly used, control charts can:

- Be used by operators for ongoing control of a process
- Help the process perform consistently and predictably
- Allow the process to achieve
  – Higher quality
  – Lower unit cost
  – Higher effective capability
- Provide a common language for discussing the performance of the process
- Distinguish special from common causes of variation, as a guide to local action or action on the system.

CHAPTER I – Section H

Effective Use and Benefits of Control Charts

Important benefits can be obtained from the effective use of control charts. The gains and benefits from the control charts are directly related to the following:

Management Philosophy: How the company is managed can directly impact the effectiveness of SPC.

The following are examples of what needs to be present:

- Focus the organization on variation reduction.
- Establish an open environment that minimizes internal competition and supports cross-functional teamwork.
- Support and fund management and employee training in the proper use and application of SPC.
- Show support and interest in the application and resulting benefits of properly applied SPC. Make regular visits and asks questions in those areas.
- Apply SPC to promote the understanding of variation in engineering processes.
- Apply SPC to management data and use the information in day-to-day decision making.

The above items support the requirements contained in ISO 9000:2000 and ISO/TS 16949:2002.

Engineering Philosophy: How engineering uses data to develop designs can and will have an influence on the level and type of variation in the finished product.

The following are some ways that engineering can show effective use of SPC:

- Focus the engineering organization on variation reduction throughout the design process; e.g., number of design changes, design for manufacturing and assembly, personnel moves, etc.
- Establish an open engineering environment that minimizes internal competition and supports cross-functional teamwork.
- Support and fund engineering management and employees training in the proper use and application of SPC.
- Apply SPC to promote the understanding of variation in engineering processes.
- Require an understanding of variation and stability in relation to measurement and the data that are used for design development.

- Support engineering changes proposed due to analysis of SPC information to aid in the reduction of variation.

Manufacturing: How manufacturing develops and operates machines and transfer systems can impact the level and type of variation in the finished product:

- Focus the manufacturing organization on variation reduction; e.g., number of different processes, impact of multi-fixture and multi-tool processes, tool and machine maintenance, etc.
- Establish an open engineering environment that minimizes internal competition and supports cross-functional teamwork.
- Support and fund manufacturing management and employees training in the proper use and application of SPC.
- Apply SPC in the understanding of variation in the manufacturing processes.
- Require an understanding of variation and stability in relation to measurement and the data that are used for process design development.
- Use the analysis of SPC information to support process changes for the reduction of variation.
- Do not release control charts to operators until the process is stable. The transfer of responsibility for the process to production should occur after the process is stable.
- Assure proper placement of SPC data for optimum use by the employees.

Quality Control: The Quality function is a critical component in providing support for an effective SPC process:

- Support SPC training for management, engineering, and employees in the organization.
- Mentor key people in the organization in the proper application of SPC.
- Assist in the identification and reduction of the sources of variation.
- Ensure optimum use of SPC data and information.

Production: Production personnel are directly related to the process and can affect process variation. They should:

- Be properly trained in the application of SPC and problem solving.
- Have an understanding of variation and stability in relation to measurement and the data that are used for process control and improvement.
- Be alert to and communicate when conditions change.
- Update, maintain and display control charts within area of responsibility.

- Interact and learn about the process from the information collected.
- Use the SPC information in real time to run the process.

Application of the concepts outlined above will result in the proper environment for the understanding and reduction of variation. Then the Plan-Do-Study-Act process can be used to further improve the process.

At a minimum, the use of SPC for process monitoring will result in the process being maintained at its current performance level. However, real improvements can be achieved when SPC is used to direct the way processes are analyzed.

Proper use of SPC can result in an organization focused on improving the quality of the product and process.

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CHAPTER II

Control Charts

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Introduction:

Control charts can be used to monitor or evaluate a process. There are basically two types of control charts, those for variables data and those for attributes data. The process itself will dictate which type of control chart to use. If the data derived from the process are of a discrete nature (e.g., go/no-go, acceptable/not acceptable) then an attributes type of chart would be used. If the data derived from the process are of a continuous nature (e.g., diameter, length) then a variables type of chart would be used. Within each chart type there are several chart combinations that can be used to further evaluate the process.

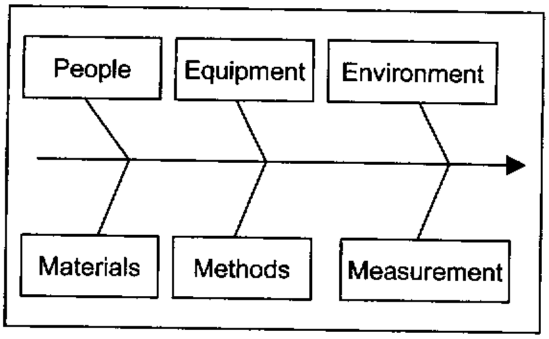
Some of the more common chart types, Average ($\bar{X}$) and Range (R) charts, Individuals (I) chart, Moving Range (MR) chart, etc., belong to the variables chart family. Charts based on count or percent data (e.g., *p*, *np*, *c*, *u*) belong to the attributes chart family.

When introducing control charts into an organization, it is important to prioritize problem areas and use charts where they are most needed. Problem signals can come from the cost control system, user complaints, internal bottlenecks, etc. The use of attributes control charts on key overall quality measures often points the way to the specific process areas that would need more detailed examination including the possible use of control charts for variables.

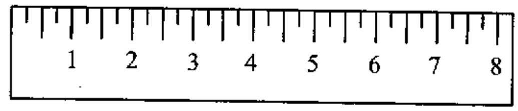
If available, variables data are always preferred as they contain more useful information than attributes data for the same amount of effort. For example you need a larger sample size for attributes than for variables data to have the same amount of confidence in the results. If the use of variables measurement systems is infeasible, the application of attributes analysis should not be overlooked.

CONTROL CHARTS TO ANALYZE THE PROCESS

Process



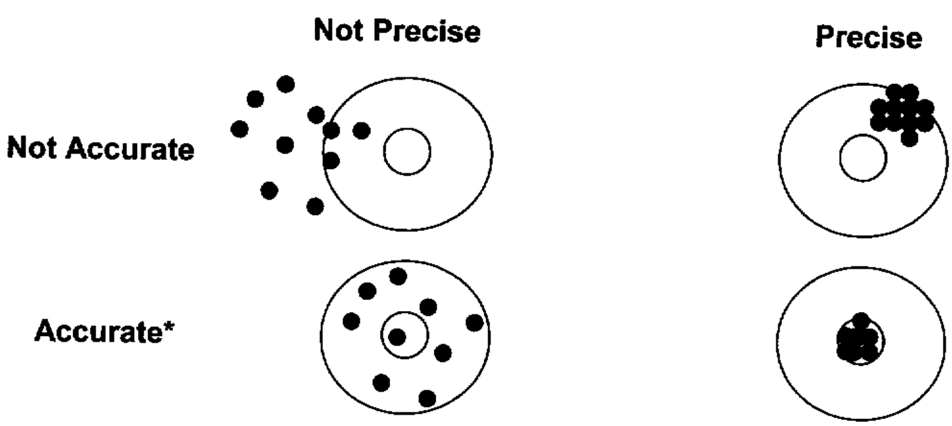
Process evaluation requires Measurements



The outcome is a Decision based on the Measurements

Outcome Example	Control Chart Examples
• Shaft O.D. (inches) • Hole distance from reference surface (mm) • Circuit resistance (ohms) • Railcar transit time (hours) • Engineering change processing time (hours)	$\overline{X}$ for the Average of the Measurement R Chart for the Ranges of the Measurement

The measurement method must produce accurate and precise results over time



*Note: Some current metrology literature defines accuracy as the lack of bias.

Figure II.1: Variables Data

Variables Control Charts

Variables control charts represent the typical application of statistical process control where the processes and their outputs can be characterized by variable measurements (see Figure II.1).

Variables control charts are particularly useful for several reasons:

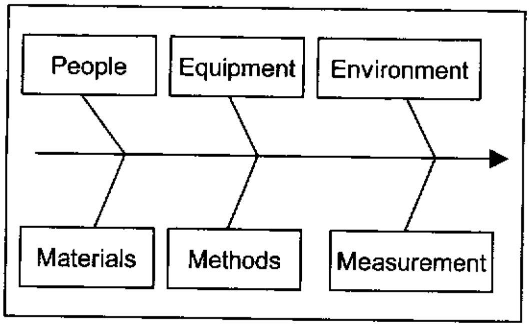
- A quantitative value (e.g., "the diameter is 16.45 mm") contains more information than a simple yes-no statement (e.g., "the diameter is within specification");
- Although collecting variables data is usually more costly than collecting attributes data (e.g., go/no-go), a decision can be reached more quickly with a smaller sample size. This can lead to lower total measurement costs due to increased efficiency;
- Because fewer parts need to be checked before making reliable decisions, the time delay between an "out-of-control" signal and corrective action is usually shorter; and
- With variables data, performance of a process can be analyzed, and improvement can be quantified, even if all individual values are within the specification limits. This is important in seeking continual improvement.

A variables chart can explain process data in terms of its process variation, piece-to-piece variation, and its process average. Because of this, control charts for variables are usually prepared and analyzed in pairs, one chart for process average and another for the process variation. The most commonly used pair are the $\overline{X}$ and R charts. $\overline{X}$ is the arithmetic average of the values in small subgroups – a measure of process average; R is the range of values within each subgroup (highest minus lowest) – a measure of process variation. However, there are a number of other control charts that may be more useful under certain circumstances.

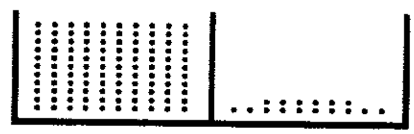
The $\overline{X}$ and R charts may be the most common charts, ***but they may not be the most appropriate for all situations.***

CONTROL CHARTS TO CLASSIFY THE PROCESS

Process



Decision is based on the Classification of the Outcome



Outcome Example	Control Chart Examples
Vehicle does not leak Lamp lights does not light Hole diameter undersized or oversized (evaluated using a go/nogo gage) Shipment to dealer correct or incorrect	*p* Chart for Proportion of Units Nonconforming *np* Chart for Number of Units Nonconforming
Bubbles in a windshield Paint imperfections on door Errors on an invoice	*c* Chart for Number of Nonconformances per Unit *u* Chart for Number of Nonconformities per Unit

The conformance criteria must be clearly defined and the procedures for deciding if those criteria are met must produce consistent results over time.

Acceptance Criteria Examples	Comment
Surface should be free from flaws	What is a flaw?
Surface should conform to master standard in color, texture, brightness and have not imperfections	Conform to what degree? Do inspectors agree? How is it measured?
Any material applied to mirror back to shall not cause visible staining of the backing	• Visible to whom? Under what conditions

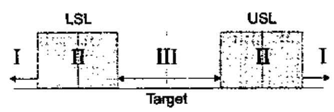
Figure II.2: Attributes Data

Attributes Control Charts

Although control charts are most often thought of in terms of variables, control charts have also been developed for attributes; see Figure II.2. Attributes data have discrete values and they can be counted for recording and analysis. With attribute analysis the data are separated into distinct categories (conforming/nonconforming, pass/fail, go/no-go, present/absent, low/medium/high). Examples include the presence of a required label, the continuity of an electrical circuit, visual analysis of a painted surface, or errors in a typed document.

Other examples are of characteristics that are measurable, but where the results are recorded in a simple yes/no fashion, such as the conformance of a shaft diameter when measured on a go/no-go gage, the acceptability of door margins to a visual or gage check, or on-time delivery performance. Control charts for attributes are important for several reasons:

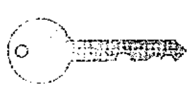
- Attributes data situations exist in any technical or administrative process, so attributes analysis techniques are useful in many applications. The most significant difficulty is to develop precise operational definitions of what is conforming.
- Attributes data are already available in many situations – wherever there are existing inspections, repair logs, sorts of rejected material, etc. In these cases, no additional effort is required for data collection. The only expense involved is for the effort of converting the data to control chart form.



- Where new data must be collected, attributes information is generally quick and inexpensive to obtain. With simple gaging (e.g., a go/no-go gage or visual standards), specialized measurement skills are often are not required. There are many occasions where specialized measurement skills are required especially when the part measured falls in the "gray" area.[16]
- Much data gathered for management summary reporting are often in attributes form and can benefit from control chart analysis. Examples include scrap rates, quality audits and material rejections. Because of the ability to distinguish between special and common cause variation, control chart analysis can be valuable in interpreting these management reports.

This manual will use conforming/nonconforming throughout attributes discussions simply because

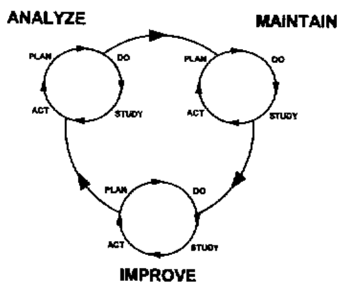
- These categories are "traditionally" used
- Organizations just starting on the path to continual improvement usually begin with these categories
- Many of the examples available in literature use these categories.



It should not be inferred that these are the only "acceptable" categories or that attributes charts cannot be used with Case 1 processes; see Chapter I, Section E.[17]

[16] See the Attribute Measurement System Study chapter in the MSA Reference Manual.
[17] See also: Montgomery (1997), Wheeler (1991, 1995), Wise and Fair (1998).

Elements of Control Charts



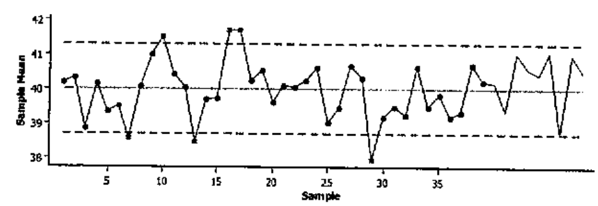
There is no single "approved" manner of displaying control charts. However the reasons for the use of control charts (see Chapter I, Section E) must be kept in mind. Any format is acceptable as long as it contains the following (see Figure II.3):

- **(A)** Appropriate scale

  The scale should be such that the natural variation of the process can be easily viewed. A scale which yields a "narrow" control chart does not enable analysis and control of the process.

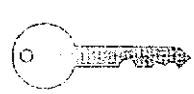
NO

YES



- **(B)** UCL, LCL

  The ability to determine outliers which signal special causes the control chart requires control limits based on the sampling distribution. Specifications limits should ***not*** be used in place of valid control limits for process analysis and control.



- **(B)** Centerline

  The control chart requires a centerline based on the sampling distribution in order to allow the determination of non-random patterns which signal special causes.

- **(C)** Subgroup sequence / timeline

  Maintaining the sequence in which the data are collected provides indications of "when" a special cause occurs and whether that special cause is time-oriented.

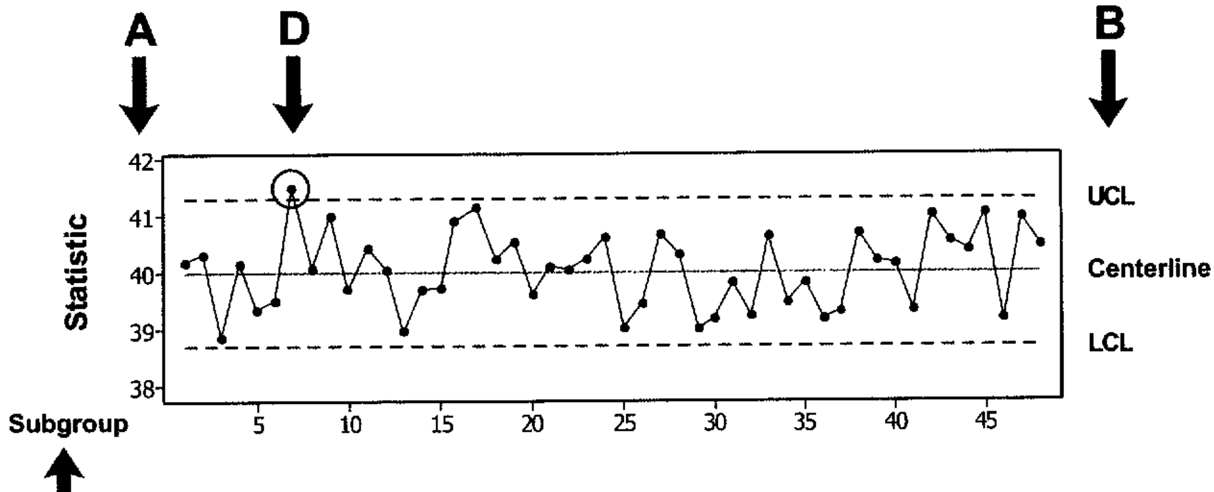
- **(D)** Identification of out-of-control plotted values

  Plotted points which are out of statistical control should be identified on the control chart. For process control the analysis for special causes and their identification should occur as each sample is plotted as well as periodic reviews of the control chart as a whole for non-random patterns.

- **(E)** Event Log

  Besides the collection, charting, and analysis of data, additional supporting information should be collected. This information should include any potential sources of variation as well as any actions taken to resolve *out-of-control signals* (OCS). This information can be recorded on the control chart or on a separate Event Log.

If there has not been any change in the process between subgroups, it is not necessary to include an entry on the process event log.



OCS	Subgroup	Event Log
	1	first shift -- new setup; insert IC27; material lot #1984
	4	second shift -- new setup; insert IC27; material lot #1931
	6	third shift -- new setup; insert IC84; material lot #1QR50
***	7	first shift -- material lot #2179, replaced bad insert
	9	third shift -- material lot #2193
	12	first shift -- material lot #1950
	23	second shift -- material lot #ZM18
	etc.	

E

Figure II.3: Elements of Control Charts

During the initial analysis of the process, knowledge of what would constitute a potential special cause for this specific process may be incomplete. Consequently, the initial information collection activities may include events which will prove out not to be special causes. Such events need not be identified in subsequent information collection activities. If initial information collection activities are not sufficiently comprehensive, then time may be wasted in identifying specific events which cause out-of-control signals.

For control charts which are included as a part of a report and for those which are maintained manually the following "header" information should be included:

- What: part/product/service name and number/identification
- Where: operation/process step information, name/identification
- Who: operator and appraiser
- How: measurement system used, name/number, units (scale)
- How many: subgroup size, uniform or by sample
- When: sampling scheme (frequency and time)

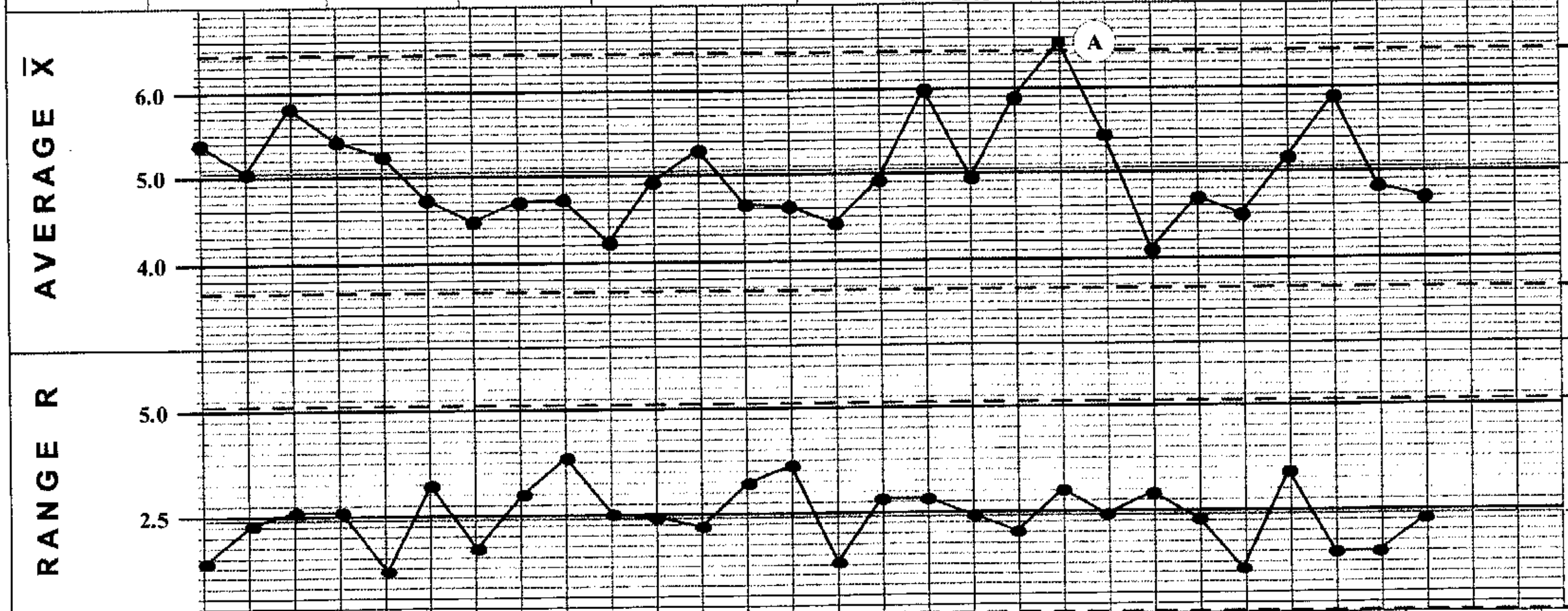
Figure II.4 shows a completed manually maintained control chart which includes all these elements

SHEET No. 43

Average and Range Chart

PART NAME	CHARACTERISTIC No.	CHARACTERISTIC	SPECIFICATION	SUBGROUP/SAMPLE SIZE	FREQUENCY	DATE START	DATE STOP
Qubic Release	LD013	DIOUEW Length	5.0 +/- 5.0	5	4 subgps/shift	xx/xx/xx	xx/xx/xx

OPERATION	MACHINE ID/FIXTURE No.	MAIN GAGE No.	BACKUP GAGE No.	MEASUREMENT SYSTEM ANALYSIS RESULTS			
VG10	FJG18	PSM05	PSM03	GRR: 13% process	Process Average $\bar{\bar{X}}$ = 5.050 Average Range $\bar{R}$ = 2.41	$UCL_{\bar{X}}$ = 6.438 $LCL_{\bar{X}}$ = 3.662	UCL_R = 5.09 LCL_R = 0.00



SUBGROUP ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14
TIME	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
DATE	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
x1	5.4	4.8	7.2	4.7	4.5	5.7	4.9	6.0	5.9	3.5	3.9	5.6	3.8	5.3
x2	6.2	6.3	4.6	4.3	5.7	4.2	3.7	3.0	3.9	5.7	4.2	4.5	3.2	6.3
x3	5.5	4.0	5.6	6.9	5.3	2.7	4.3	5.3	6.5	3.2	6.2	5.4	4.9	2.7
x4	5.1	6.0	5.1	6.8	5.4	5.1	4.1	3.9	2.7	3.6	6.3	4.4	5.0	4.4
x5	4.8	4.1	6.6	4.4	5.3	5.9	5.4	5.3	4.6	5.2	4.0	6.6	6.4	4.5
SUM OF INDIVIDUALS	27.0	25.2	29.1	27.1	26.2	23.6	22.4	23.5	23.6	21.2	24.6	26.5	23.3	23.2
AVERAGE - X	5.40	5.04	5.82	5.42	5.24	4.72	4.48	4.70	4.72	4.24	4.92	5.30	4.66	4.64
RANGE - R	1.4	2.3	2.6	2.6	1.2	3.2	1.7	3.0	3.8	2.5	2.4	2.2	3.2	3.6

SUBGROUP ID	15	16	17	18	19	20	21	22	23	24	25	26	27	28
TIME	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
DATE	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
x1	3.9	6.5	7.5	6.4	5.6	6.3	4.6	3.5	5.2	4.2	5.2	6.4	4.5	5.4
x2	4.7	4.7	6.4	4.2	6.7	8.1	5.8	5.0	6.1	4.4	5.1	6.4	5.2	5.6
x3	5.2	3.7	4.7	4.6	4.9	5.1	4.7	5.9	3.8	5.0	3.5	5.9	4.9	3.3
x4	4.3	4.2	6.7	5.5	5.4	5.5	5.1	3.0	4.3	5.0	6.9	4.9	4.0	5.4
x5	4.1	5.5	4.7	4.0	6.9	7.7	7.0	3.2	4.1	3.9	5.2	5.9	5.5	3.8
SUM OF INDIVIDUALS	22.2	24.6	30.0	24.7	29.5	32.7	27.2	20.6	23.5	22.5	25.9	29.5	24.1	23.5
AVERAGE - X	4.44	4.92	6.00	4.94	5.90	6.54	5.44	4.12	4.70	4.50	5.18	5.90	4.82	4.70
RANGE - R	1.3	2.8	2.8	2.4	2.0	3.0	2.4	2.9	2.3	1.1	3.4	1.5	1.5	2.3

Figure II.4a: Sample Control Chart (Front side)

OCS	Subgroup	Event Log
	1	S1 -- new setup; insert MH18; material lot #121950
	2	S1 -- n/c
	3	S1 -- material lot #081984
	4	S1 -- n/c
	5	S2 -- material lot #081950
	6	S2 -- new setup; insert DA14; continue material lot #081950
	7	S2 -- material lot #111951
	8	S2 -- n/c
	9	S3 -- new setup; insert DM19; material lot #111952
	10	S3 -- replaced broken insert with DM23; continue material lot
	11	S3 -- material lot #111953
	12	S3 -- n/c
	13	S1 -- new setup; insert JK10; material lot #111954
	14	S1 -- n/c
	15	S1 -- material lot #111955
	16	S1 -- n/c
	17	S2 -- material lot #111956
	18	S2 -- backup operator -- D.A.
	19	S2 -- material lot #111957
A	20	S2 -- bad material -- stopped production; red tagged material
		sequestered production from lot change.
		sorted production -- found 21 O/S parts
	21	S3 -- new setup; insert JK28; material lot #111958
	22	S3 -- n/c
	23	S3 -- material lot #081943
	24	S3 -- n/c
	25	S1 -- new setup; insert GG16; material lot #031942
	26	S1 -- n/c
	27	S1 -- material lot #111940
	28	S1 -- n/c

Note: Sx indicates the shift; n/c indicates no changes in the process.

Figure II.4b: Sample Control Chart (back side) – Event Log

CHAPTER II - Section A

Control Chart Process

Preparatory Steps

Before control charts can be used, several preparatory steps should be taken:

✓ Establish an environment suitable for action.

✓ Define the process.

✓ Determine the features or characteristics to be charted based on:

- The customer's needs.
- Current and potential problem areas.
- Correlation between characteristics.

Caution

Correlation between variables does not imply a causal relationship. In the absence of process knowledge, a designed experiment may be needed to verify such relationships and their significance.

✓ Define the characteristic.

The characteristic must be operationally defined so that results can be communicated to all concerned in ways that have the same meaning today as yesterday. This involves specifying what information is to be gathered, where, how, and under what conditions.

An operational definition describes the characteristic that is to be evaluated and whether the characteristic is qualitative (discrete) or quantitative (continuous). Attributes control charts would be used to monitor and evaluate discrete variables whereas variables control charts would be used to monitor and evaluate continuous variables.

✓ Define the measurement system.

Total process variability consists of part-to-part variability *and* measurement system variability. It is very important to evaluate the effect of the measurement system's variability on the overall process variability and determine whether it is acceptable. The measurement performance must be predictable in terms of accuracy, precision and stability.

Periodic calibration is not enough to validate the measurement system's capability for its intended use. In addition to being calibrated, the measurement system must be evaluated in terms of its suitability for the intended use.

For more detail on this subject see the *Measurement Systems Analysis* (MSA) Reference Manual. The definition of the measurement system will determine what type of chart, variables or attributes, is appropriate.

✓ Minimize unnecessary variation.

Unnecessary external causes of variation should be reduced before the study begins. This could simply mean watching that the process is being operated as intended. The purpose is to avoid obvious problems that could and should be corrected without use of control charts. This includes process adjustment or over control. In all cases, a process event log may be kept noting all relevant events such as tool changes, new raw material lots, measurement system changes, etc. This will aid in subsequent process analysis.

✓ Assure selection scheme is appropriate for detecting expected special causes.

WARNING: Even though convenience sampling and/or haphazard sampling is often thought of as being random sampling, it is not. If one assumes that it is, and in reality it is not, one carries an unnecessary risk that may lead to erroneous and or biased conclusions.

For more details see Chapter I, Section H.

Control Chart Mechanics

The steps to using control charts are:

1. Data Collection
2. Establish Control Limits
3. Interpret for Statistical Control
4. Extend Control Limits for ongoing control (see Figure II.5)

Data Collection

Control charts are developed from measurements of a particular characteristic or feature of the process. These measurements are combined into a (control) statistic (e.g., average, median, range, standard deviation, individual) which describes an attribute of the process distributional form. The measurement data are collected from individual samples from a process stream. The samples are collected in subgroups and may consist of one or more pieces. In general, a larger subgroup size makes it easier to detect small process shifts.

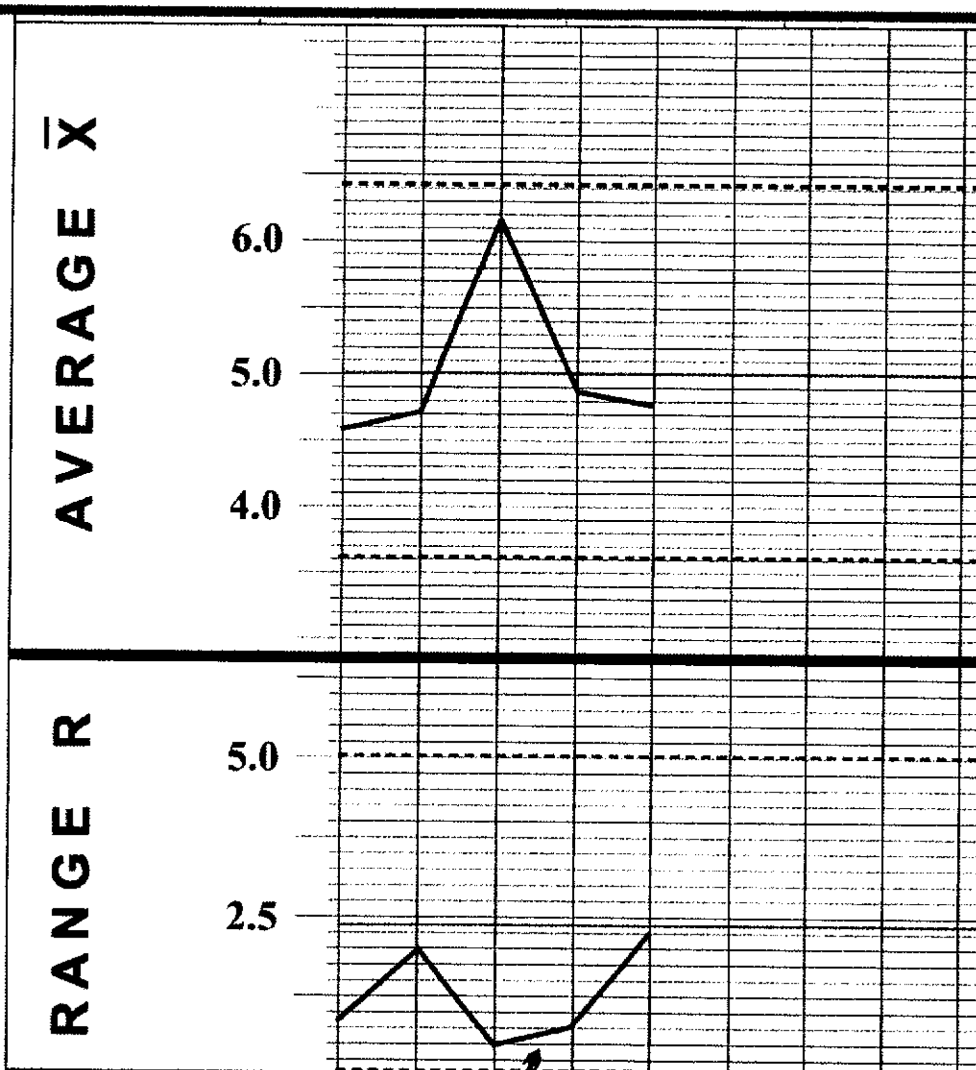
Create a Sampling Plan

For control charts to be effective the sampling plan should define *rational* subgroups. A rational subgroup is one in which the samples are selected so that the chance for variation due to special causes occurring *within* a subgroup is minimized, while the chance for special cause variation *between* subgroups is maximized. The key item to remember when developing a sampling plan is that the variation between subgroups is going to be compared to the variation within subgroups. Taking consecutive samples for the subgroups minimizes the opportunity for the process to change and should minimize the *within-subgroup* variation. The sampling frequency will determine the opportunity the process has to change between subgroups.

The variation within a subgroup represents the piece-to-piece variation over a short period of time.[18] Any significant variation between subgroups would reflect changes in the process that should be investigated for appropriate action.

Subgroup Size – The type of process under investigation dictates how the subgroup size is defined. As stated earlier, a larger subgroup size makes it easier to detect small process shifts. The team responsible has to determine the appropriate subgroup size. If the expected shift is relatively small, then a larger subgroup size would be needed compared to that required if the anticipated shift is large.

[18] See also Appendix A.



SUBGROUP ID									
TIME									
DATE	6/17								
x1	4.2	5.2	6.4	4.5	5.4				
x2	4.4	5.1	6.4	5.2	5.6				
x3	5.0	3.5	5.9	4.9	3.3				
x4									
x5									
SUM OF INDIVIDUALS	13.6	13.8	18.7	14.6	14.3				
AVERAGE - $\bar{X}$	4.53	4.60	6.23	4.87	4.77				
RANGE - R	0.8	1.7	0.5	0.7	2.3				

Figure II.5: Extending Control Limits

The subgroup size should remain constant but there may be situations where the subgroup size varies within a single control chart. The calculation of the control limits depends on the subgroup size and if one varies the subgroup size the control limits will change for that subgroup. There are other techniques that deal with variable subgroup sizes; for example, see Montgomery (1997) and Grant and Leavenworth (1996).

Subgroup Frequency – The subgroups are taken sequentially in time, e.g., once every 15 minutes or twice per shift. The goal is to detect changes in the process over time. Subgroups should be collected often enough, and at appropriate times so that they can reflect the potential opportunities for change. The potential causes of change could be due to work-shift differences, relief operators, warm-up trends, material lots, etc.

Number of Subgroups – The number of subgroups needed to establish control limits should satisfy the following criterion: enough subgroups should be gathered to assure that the major sources of variation which can affect the process have had an opportunity to appear. Generally, 25 or more subgroups containing about 100 or more individual readings give a good test for stability and, if stable, good estimates of the process location and spread. This number of subgroups ensures that the effect of any extreme values in the range or standard deviation will be minimized.

In some cases, existing data may be available which could accelerate this first stage of the study. However, they should be used only if they are recent and if the basis for establishing subgroups is clearly understood. Before continuing, a rational sampling plan must be developed and documented.

Sampling Scheme – If the special causes affecting the process can occur unpredictably, the appropriate sampling scheme is a random (or probability) sample. A random sample is one in which every sample point (rational subgroup) has the same chance (probability) of being selected. A random sample is systematic and planned; that is, all sample points are determined before any data are collected. For special causes that are known to occur at specific times or events, the sampling scheme should utilize this knowledge. Haphazard sampling or convenience sampling not based on the expected occurrence of a specific special cause should be avoided since this type of sampling provides a false sense of security; it can lead to a biased result and consequently a possible erroneous decision.

Whichever sampling scheme is used all sample points should be determined before any data are collected (see Deming (1950) and Gruska (2004)).

NOTE: For a discussion about rational subgrouping and the effect of subgrouping on control chart interpretation see Appendix A.

Control Chart Setup

A control chart will have sections for:

- ✓ Header information including the description of the process and sampling plan.
- ✓ Recording/displaying the actual data values collected.

  This should also include the date & time or other subgroup identification.

- ✓ For interim data calculations (optional for automated charts).

  This should also include a space for the calculations based on the readings and the calculated control statistic(s).

- ✓ For plotting each of the control statistics being analyzed.

  The value for the control statistic is usually plotted on the vertical scale and the horizontal scale is the sequence in time. The data values and the plot points for the control statistic should be aligned vertically. The scale should be broad enough to contain all the variation in the control statistic. A guideline is that the initial scale could be set to twice the difference between the (expected) maximum and minimum values.

- ✓ To log observations.

  This section should include details such as process adjustments, tooling changes, material changes, or other events which may affect the variability of the process.

Record Raw Data

- ✓ Enter the individual values and the identification for each subgroup.
- ✓ Log any pertinent observation(s).

Calculate Sample Control Statistic(s) for Each Subgroup

The control statistics to be plotted are calculated from the subgroup measurement data. These statistics may be the sample mean, median, range, standard deviation, etc. Calculate the statistics according to the formulae for the type of chart that is being used.

Plot the Control Statistic(s) on the Control Charts

Plot the control statistic on the chart. Make sure that the plot points for the corresponding control statistics are aligned vertically. Connect the points with lines to help visualize patterns and trends.

The data should be reviewed while they are being collected in order to identify potential problems. If any points are substantially higher or lower than the others, confirm that the calculations and plots are correct and log any pertinent observations.

Establish Control Limits

Control limits are defined by the natural variation of the control statistic. They define a range of values that the control statistic could randomly fall within, given there is only common cause to the variation. If the average of two different subgroups from the same process is calculated, it is reasonable to expect that they will be about the same. But since they were calculated using different parts, the two averages are not expected to be identical. Even though the two averages are different, there is a limit to how different they are expected to be, *due to random chance*. This defines the location of the control limits.

This is the basis for all control chart techniques. If the process is stable (i.e., having only *common cause* variation), then there is a high probability that for any subgroup sample the calculated control statistic will fall within the control limits. If the control statistic exceeds the control limits then this indicates that a *special cause* variation may be present.

There are two phases in statistical process control studies.

1. The first is identifying and eliminating the special causes of variation in the process. The objective is to stabilize the process. A stable, predictable process is said to be *in statistical control.*
2. The second phase is concerned with predicting future measurements thus verifying ongoing process stability. During this phase, data analysis and reaction to special causes is done in real time. Once stable, the process can be analyzed to determine if it is capable of producing what the customer desires.

Identify the centerline and control limits of the control chart

To assist in the graphical analysis of the plotted control statistics, draw lines to indicate the location estimate (centerline) and control limits of the control statistic on the chart.

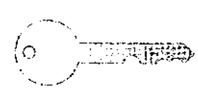
In general, to set up a control chart calculate:

- ✓ Centerline,
- ✓ Upper Control Limit (UCL),
- ✓ Lower Control Limit (LCL).

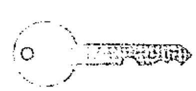
See Chapter II, Section C, for the formulas.

Interpret for Statistical Control

If the process has no special causes affecting its variability, then the control statistics will fall between the control limits in a random fashion (i.e., no patterns will be evident).



Special causes can affect either the process location (e.g., average, median) or the variation (e.g., range, standard deviation) or both. The objective of control chart analysis is to identify any evidence that the process variability or the process location is not operating at a constant level – that one or both are out of statistical control – and to take appropriate action.



In the subsequent discussion, the Average will be used for the location control statistic and the Range for the variation control statistic. The conclusions stated for these control statistics also apply equally to the other possible control statistics.

Since the control limits of the location statistic are dependent on the variation statistic, the variation control statistic should be analyzed first for stability. The variation and location statistics are analyzed separately, but comparison of patterns between the two charts may sometimes give added insight into special causes affecting the process

A process cannot be said to be stable (in statistical control) unless both charts have no out-of-control conditions (indications of special causes).

Analyze the Data Plots on the Range Chart

Since the ability to interpret either the subgroup ranges or subgroup averages depends on the estimate of piece-to-piece variability, the R chart is analyzed first. The data points are compared with the control limits, for points out of control or for unusual patterns or trends (see Chapter II, Section D)

Find and Address Special Causes (Range Chart)

For each indication of a special cause in the range chart data, conduct an analysis of the process operation to determine the cause and improve process understanding; correct that condition, and prevent it from recurring. The control chart itself should be a useful guide in problem analysis, suggesting when the condition may have began and how long it continued. However, recognize that not all special causes are negative; some special causes can result in positive process improvement in terms of decreased variation in the range – those special causes should be assessed for possible institutionalization within the process, where appropriate.

Timeliness is important in problem analysis, both in terms of minimizing the production of inconsistent output, and in terms of having fresh evidence for diagnosis. For instance, the appearance of a single point beyond the control limits is reason to begin an immediate analysis of the process. A process event log may also be a helpful source of information in terms of identifying special causes of variation.

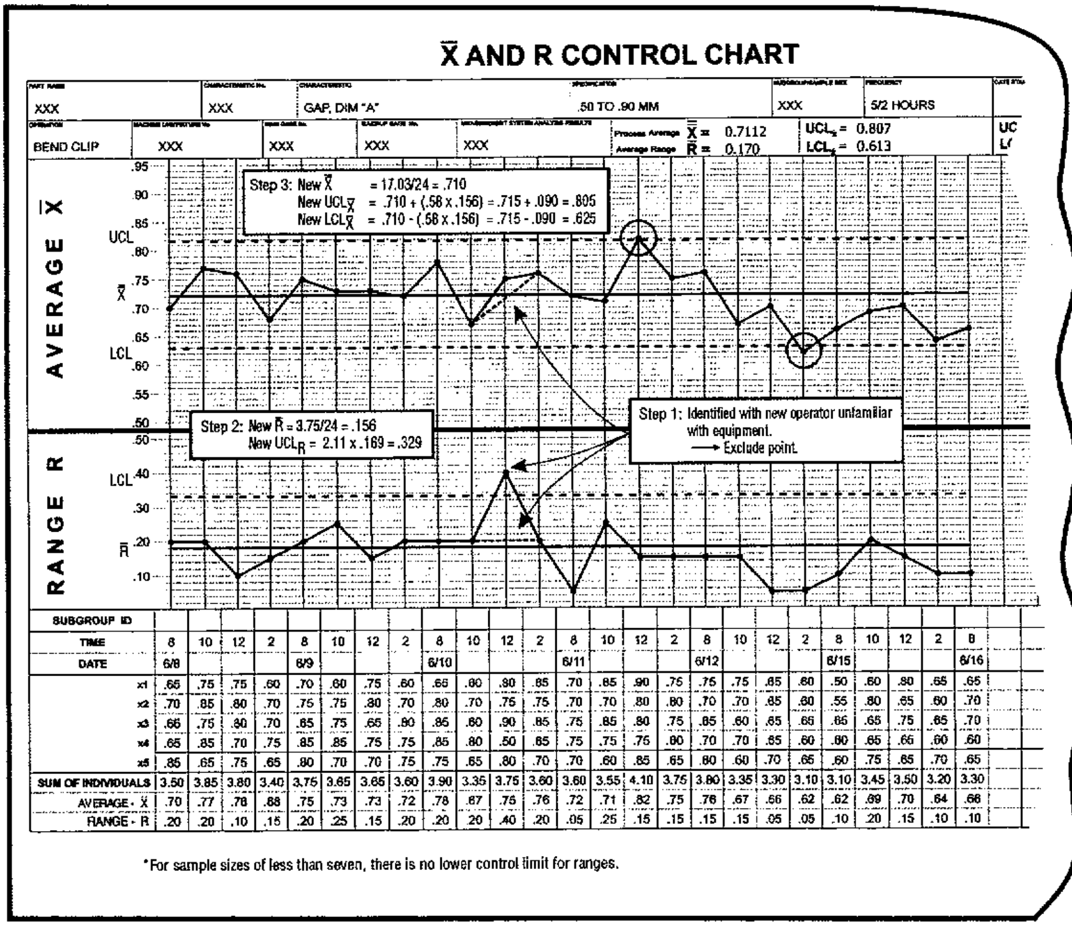


Figure II.6: Control Limits Recalculation

It should be emphasized that problem solving is often the most difficult and time-consuming step. Statistical input from the control chart can be an appropriate starting point, but other methods such as Pareto charts, cause and effect diagrams, or other graphical analysis can be helpful (see Ishikawa (1976)). Ultimately, however, the explanations for behavior lie within the process and the people who are involved with it. Thoroughness, patience, insight and understanding will be required to develop actions that will measurably improve performance.

Recalculate Control Limits (Range Chart)

When conducting an initial process study or a reassessment of process capability, the control limits should be recalculated to exclude the effects of out-of-control periods for which process causes have been clearly identified and removed or institutionalized. Exclude all subgroups affected by the special causes that have been identified and removed or institutionalized, then recalculate and plot the new average range ($\overline{R}$) and control limits. Confirm that all range points show control when compared to the new limits; if not, repeat the identification, correction, recalculation sequence.

If any subgroups were dropped from the R chart because of identified special causes, they should also be excluded from the $\overline{X}$ chart. The revised $\overline{R}$ and $\overline{\overline{X}}$ should be used to recalculate the trial control limits for averages, $\overline{\overline{X}} \pm A_2\overline{R}$ (see Figure II.6).

NOTE: The exclusion of subgroups representing unstable conditions is not just "throwing away bad data." Rather, by excluding the points affected by known special causes, there is a better estimate of the background level of variation due to common causes. This, in turn, gives the most appropriate basis for the control limits to detect future occurrences of special causes of variation. Be reminded, however, that the process must be changed so the special cause will not recur (if undesirable) as part of the process.

Find and Address Special Causes (Average Chart)

Once the special cause which affect the variation (Range Chart) have been identified and their effect have been removed, the Average Chart can be evaluated for special causes. In Figure II.6 the new control limits for the averages indicate that two samples are out of control.

For each indication of an out-of-control condition in the average chart data, conduct an analysis of the process operation to determine the reason for the special cause; correct that condition, and prevent it from recurring. Use the chart data as a guide to when such conditions began and how long they continued. Timeliness in analysis is important, both for diagnosis and to minimize inconsistent output. Again, be aware that not all special causes need be undesirable (see Chapter I, Section E and Chapter II, Section B).

Problem solving techniques such as Pareto analysis and cause-and-effect analysis can help. (Ishikawa (1976)).

Recalculate Control Limits (Average Chart)

When conducting an initial process study or a reassessment of process capability, exclude any out-of-control points for which special causes have been found and removed; recalculate and plot the process average and control limits. Confirm that all data points show control when compared to the new limits; if not, repeat the identification, correction, recalculation sequence.

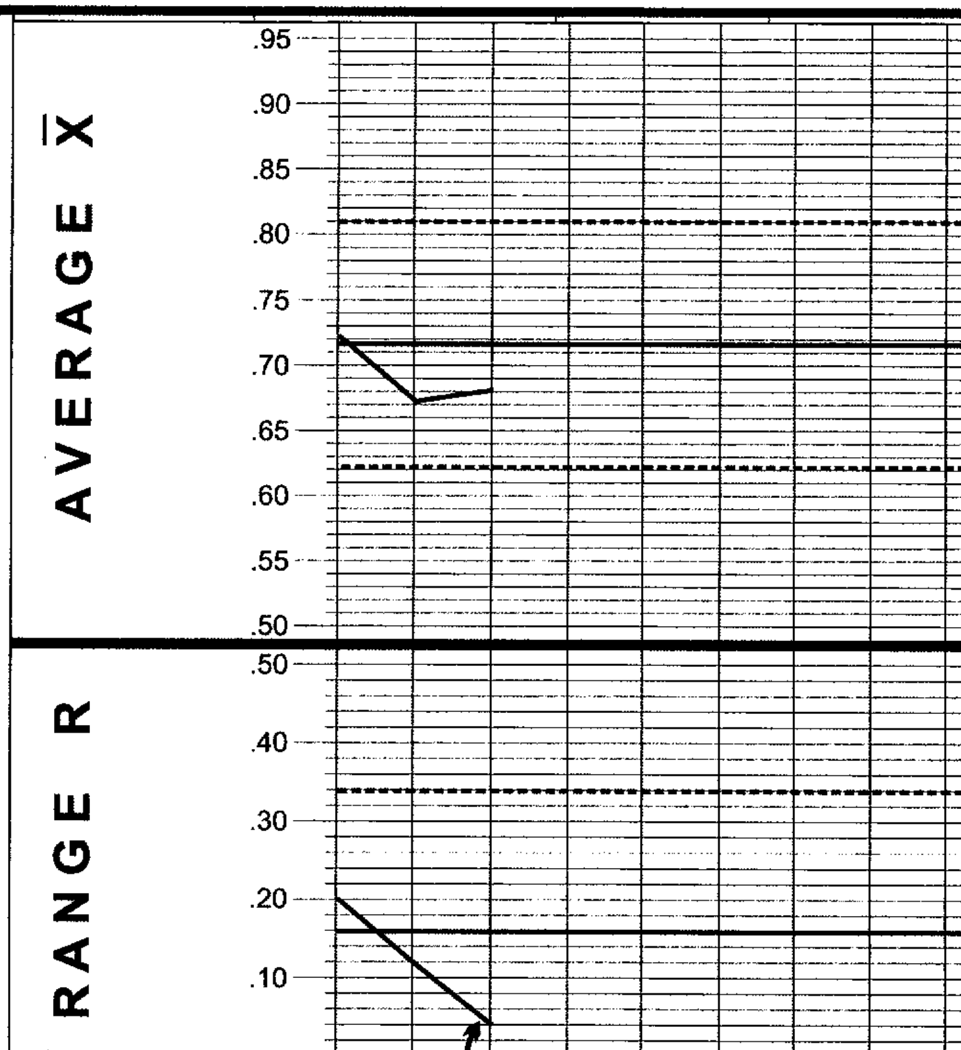
Final Comments

The preceding discussions were intended to give a functional introduction to control chart analysis. Even though these discussions used the Average and Range Charts, the concepts apply to all control chart approaches.

Furthermore, there are other considerations that can be useful to the analyst. One of the most important is the reminder that, even with processes that are in statistical control, the probability of getting a false signal of a special cause on any individual subgroup increases as more data are reviewed.

While it is wise to investigate all signals as possible evidence of special causes, it should be recognized that they may have been caused by the system and that there may be no underlying local process problem. If no clear evidence of a special cause is found, any "corrective" action will probably serve to increase, rather than decrease, the total variability in the process output.

For further discussion of interpretation, tests for randomness in data, and problem-solving, see AT&T (1984), Duncan (1986), Grant and Leavenworth (1996), Juran and Godfrey (1999), Charbonneau and Gordon (1978), Ishikawa (1976), Wheeler (1991, 1995), and Ott (2000).



SUBGROUP ID									
TIME									
DATE	6/17								
x1	.80	.60	.65						
x2	.75	.70	.70						
x3	.60	.70	.70						
x4									
x5									
SUM OF INDIVIDUALS	2.15	2.00	2.05						
AVERAGE - $\bar{X}$	.72	.67	.68						
RANGE - R	.20	.10	.05						

Figure II.7: Extend Control Limits for Ongoing Control

Extend Control Limits for Ongoing Control

When the initial (or historical) data are consistently contained within the trial control limits, extend the limits to cover future periods. It might be desirable here to adjust the process to the target if the process center is off target. These limits would be used for ongoing monitoring of the process, with the operator and local supervision responding to signs of out-of-control conditions on either the location and variation $\bar{X}$ or R chart with prompt action (see Figure II.7).

A change in the subgroup sample size would affect the expected average range and the control limits for both ranges and averages. This situation could occur, for instance, if it were decided to take smaller samples more frequently, so as to detect large process shifts more quickly without increasing the total number of pieces sampled per day. To adjust central lines and control limits for a new subgroup sample size, the following steps should be taken:

- Estimate the process standard deviation (the estimate is shown as $\hat{\sigma}$ –"sigma hat").[19] Using the existing subgroup size calculate: $\hat{\sigma}_C = \bar{R}/d_2$ where $\bar{R}$ is the average of the subgroup ranges (for periods with the ranges in control) and d_2 is a constant varying by sample size n, the number of samples in a subgroup, as shown in the partial table below, taken from Appendix E:

n	2	3	4	5	6	7	8	9	10
d_2	1.13	1.69	2.06	2.33	2.53	2.70	2.85	2.97	3.08

- Using the table factors based on the new subgroup size, calculate the new range and control limits:

$$\bar{R}_{new} = \hat{\sigma}_C \cdot d_2$$

Plot these new control limits on the chart as the basis for ongoing process control. As long as the process remains in control for both averages and ranges, the ongoing limits can be extended for additional periods. If, however, there is evidence that the process average or range has changed (in either direction), the cause should be determined and, if the change is justifiable, control limits should be recalculated based on current performance.

[19] This manual will distinguish between the estimated standard deviation due to the within-subgroup variation and the total variation by using the subscripts "*C*" and "*P*", respectively.

Final Concepts on "Control" – For Further Consideration

"A perfect state of control is never attainable in a production process. The goal of the process control charts is not perfection, but a reasonable and economical state of control. For practical purposes, therefore, a controlled process is not one where the chart never goes out of control. If a chart never went out of control we would seriously question whether that operation should be charted. For shop purposes a controlled process is considered to be one where only a small percentage of the points go out of control and where out-of-control points are followed by proper action[20]." See also Figure II.8.

Obviously, there are different levels or degrees of statistical control. The definition of control used can range from mere outliers (beyond the control limits), through runs, trends and stratification, to full zone analysis. As the definition of control used advances to full zone analysis, the likelihood of finding lack of control increases (for example, a process with no outliers may demonstrate lack of control through an obvious run still within the control limits). For this reason, the definition of control used should be consistent with your ability to detect this at the point of control and should remain the same within one time period, within one process. Some suppliers may not be able to apply the fuller definitions of control on the floor on a real-time basis due to immature stages of operator training or lack of sophistication in the operator's ability. The ability to detect lack of control at the point of control on a real-time basis is an advantage of the control chart. Over-interpretation of the data can be a danger in maintaining a true state of economical control.

[20] AT&T (1984)

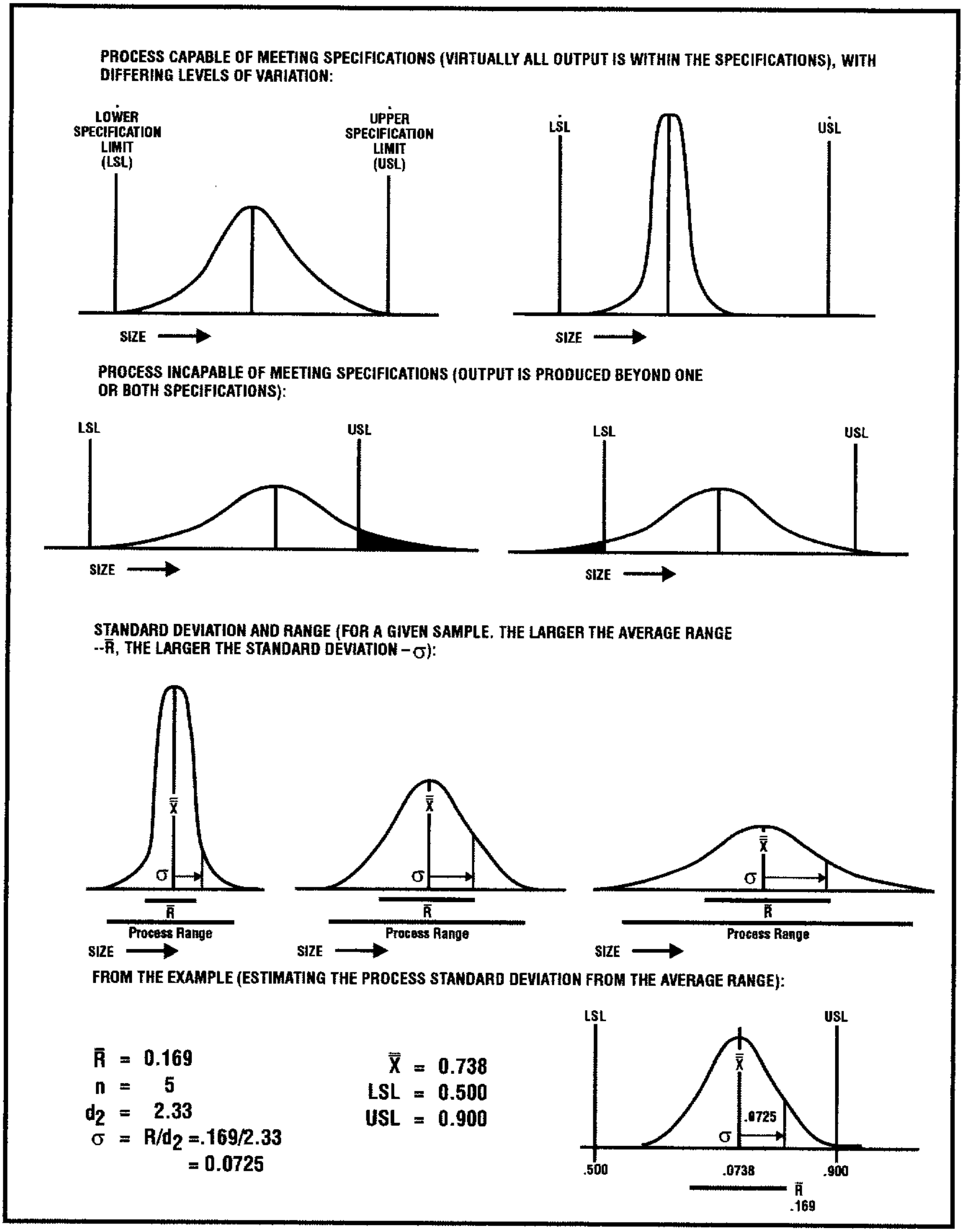


Figure II.8: Process Variation Relative to Specification Limits

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CHAPTER II - Section B

Defining "Out-of-Control" Signals

Point Beyond a Control Limit.

The presence of one or more points beyond either control limit is primary evidence of special cause variation at that point. This special cause could have occurred prior to this point.

Since points beyond the control limits would be rare if only variation from common causes were present, the presumption is that a special cause has accounted for the extreme value. Therefore, any point beyond a control limit is a signal for analysis of the operation for the special cause. Mark any data points that are beyond the control limits for investigation and corrective action based on when that special cause actually started.

A point outside a control limit is generally a sign of one or more of the following:

- The control limit or plot point has been miscalculated or misplotted.
- The piece-to-piece variability or the spread of the distribution has increased (i.e., worsened), either at that one point in time or as part of a trend.
- The measurement system has changed (e.g., a different appraiser or instrument).
- The measurement system lacks appropriate discrimination.

For charts dealing with the spread, a point below the lower control limit is generally a sign of one or more of the following:

- The control limit or plot point is in error.
- The spread of the distribution has decreased (i.e., becomes better).
- The measurement system has changed (including possible editing or alteration of the data).

A point beyond either control limit is generally a sign that the process has shifted either at that one point or as part of a trend (see Figure II.9).

When the ranges are in statistical control, the process spread – the within-subgroup variation – is considered to be stable. The averages can then be analyzed to see if the process location is changing over time. Since control limits for $\bar{X}$ are based upon the amount of variation in the ranges, then if the averages are in statistical control, their variation is related to the amount of variation seen in the ranges – the common-cause variation of the system. If the averages are not in control, some special causes of variation are making the process location unstable.

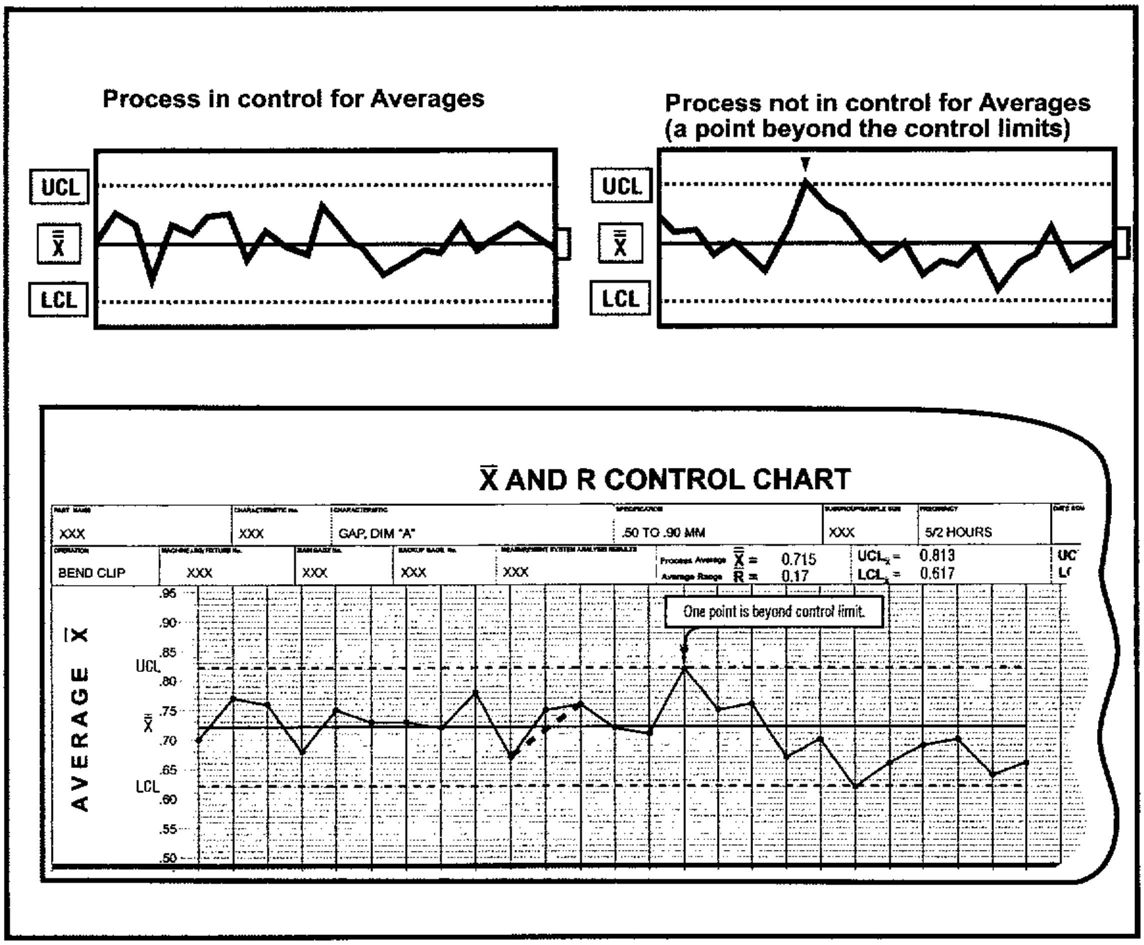


Figure II.9: Points Beyond Control Limits

Patterns or Trends Within the Control Limits

The presence of unusual patterns or trends, even when all ranges are within the control limits, can be evidence of the influence of a special cause during the period of the pattern or trend. This could give the first warning of an unfavorable condition which should be corrected. Conversely, certain patterns or trends could be favorable and should be studied for possible permanent improvement of the process. Comparison of patterns between the range and average charts may give added insight.

There are situations where an "out-of-control pattern" may be a bad event for one process and a good event for another process. An example of this is that in an $\bar{X}$ and R chart a series of 7 or more points on one side of the centerline may indicate an out-of-control situation. If this happened in a p chart, the process may actually be improving if the series is below the average line (less nonconformances are being produced). So in this case the series is a good thing – if we identify and retain the cause.

Runs

Runs – Each of the following are signs that a process shift or trend has begun:

- 7 points in a row on one side of the $\overline{\overline{X}}$ or $\overline{R}$.
- 7 points in a row that are consistently increasing (equal to or greater than the preceding points), or consistently decreasing.

Mark the point that prompts the decision; it may be helpful to extend a reference line back to the beginning of the run. Analysis should consider the approximate time at which it appears that the trend or shift first began.

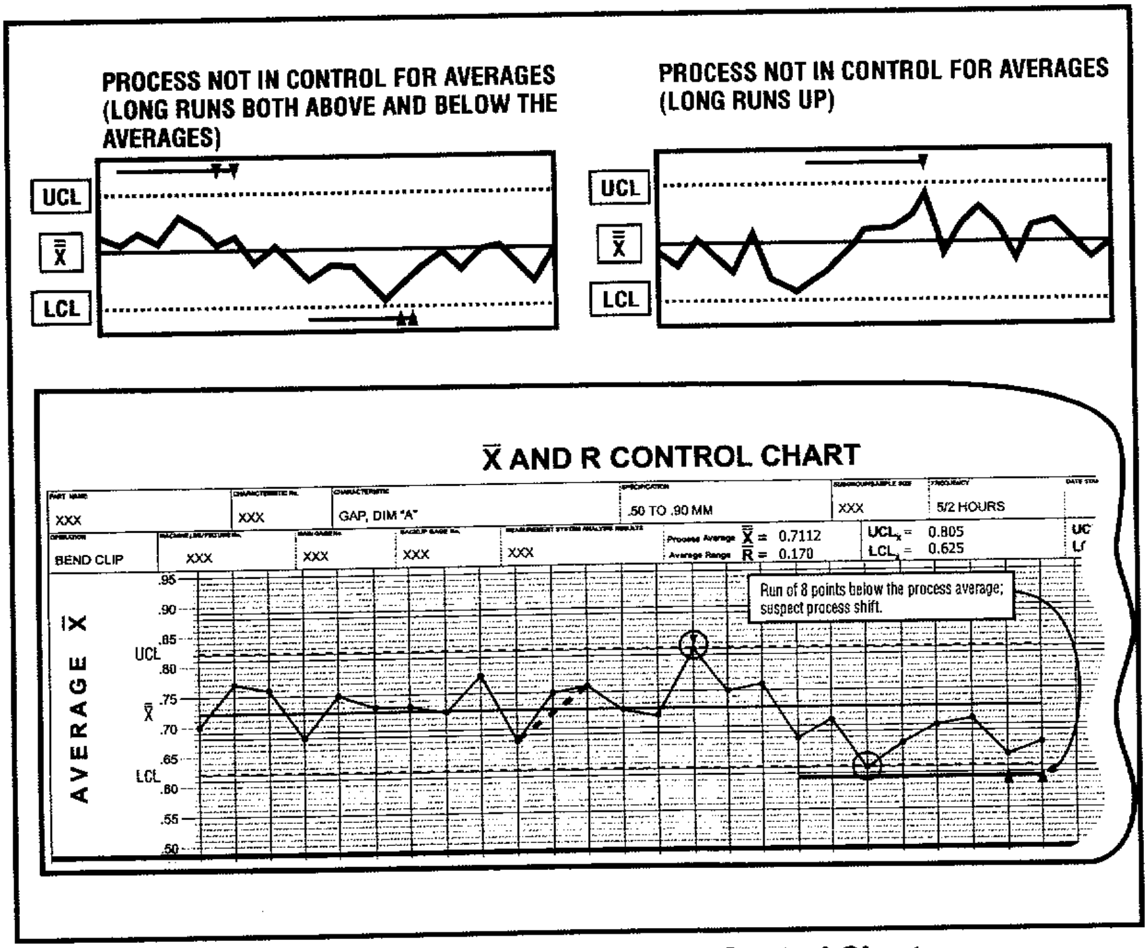
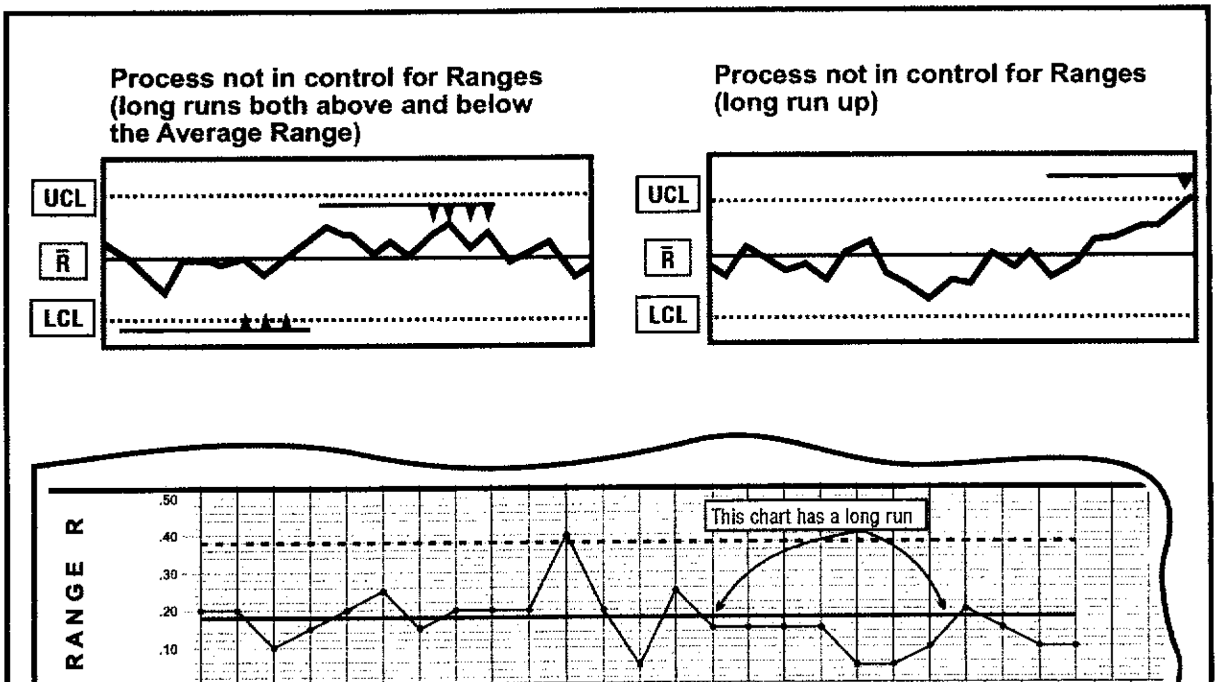


Figure II.10: Runs in an Average Control Chart

A run above the average range, or a run up, signifies one or both of the following:

- ✓ Greater spread in the output values, which could be from an irregular cause (such as equipment malfunction or loose fixturing) or from a shift in one of the process elements (e.g., a new, less uniform raw material lot).
- ✓ A change in the measurement system (e.g., new inspector or gage).



SUBGROUP ID													
TIME	8	10	12	2	8	10	12	2	8	10	12	2	8
DATE	6/8				6/9				6/10				6/11
x1	.85	.75	.75	.60	.70	.60	.75	.60	.65	.60	.80	.85	.70
x2	.70	.85	.80	.70	.75	.75	.80	.70	.80	.70	.75	.75	.70
x3	.65	.75	.80	.70	.65	.75	.65	.80	.85	.60	.90	.85	.75
x4	.65	.85	.70	.75	.85	.85	.75	.75	.85	.80	.50	.65	.75
x5	.85	.65	.75	.65	.80	.70	.70	.75	.75	.65	.80	.70	.70
SUM OF INDIVIDUALS	3.50	3.85	3.80	3.40	3.75	3.65	3.65	3.60	3.90	3.35	3.75	3.60	3.60
AVERAGE - X̄	.70	.77	.76	.68	.75	.73	.73	.72	.78	.67	.75	.76	.72
RANGE - R	.20	.20	.10	.15	.20	.25	.15	.20	.20	.20	.40	.20	.05

SUBGROUP ID												
TIME	10	12	2	8	10	12	2	8	10	12	2	8
DATE				6/12				6/15				6/16
x1	.65	.90	.75	.75	.75	.65	.60	.50	.60	.80	.65	.65
x2	.70	.80	.80	.70	.70	.65	.60	.55	.80	.65	.60	.70
x3	.85	.80	.75	.85	.60	.65	.65	.65	.65	.75	.65	.70
x4	.75	.75	.80	.70	.70	.65	.60	.60	.65	.65	.60	.60
x5	.60	.85	.65	.80	.60	.70	.65	.60	.75	.65	.70	.65
SUM OF INDIVIDUALS	3.55	4.10	3.75	3.80	3.35	3.30	3.10	3.10	3.45	3.50	3.20	3.30
AVERAGE - X̄	.71	.82	.75	.76	.67	.66	.62	.62	.69	.70	.64	.66
RANGE - R	.25	.15	.15	.15	.15	.05	.05	.10	.20	.15	.10	.10

Figure II.11: Runs in a Range Control Chart

A run below the average range, or a run down, signifies one or both of the following:

- ✓ Smaller spread in output values, which is usually a good condition that should be studied for wider application and process improvement.
- ✓ A change in the measurement system, which could mask real performance changes.

NOTE: As the subgroup size (n) becomes smaller (5 or less), the likelihood of runs below $\overline{R}$ increases, so a run length of 8 or more could be necessary to signal a decrease in process variability.

A run relative to the process average is generally a sign of one or both of the following:

- ✓ The process average has changed – and may still be changing.
- ✓ The measurement system has changed (drift, bias, sensitivity, etc.).

Obvious Nonrandom Patterns

In addition to the presence of points beyond control limits or long runs, other distinct patterns may appear in the data that give clues to special causes. Care should be taken not to over-interpret the data, since even random (i.e., common cause) data can sometimes give the illusion of nonrandomness (i.e., special causes). Examples of nonrandom patterns could be obvious trends (even though they did not satisfy the runs tests), cycles, the overall spread of data points within the control limits, or even relationships among values within subgroups (e.g., the first reading might always be the highest). One test for the overall spread of subgroup data points is described below.

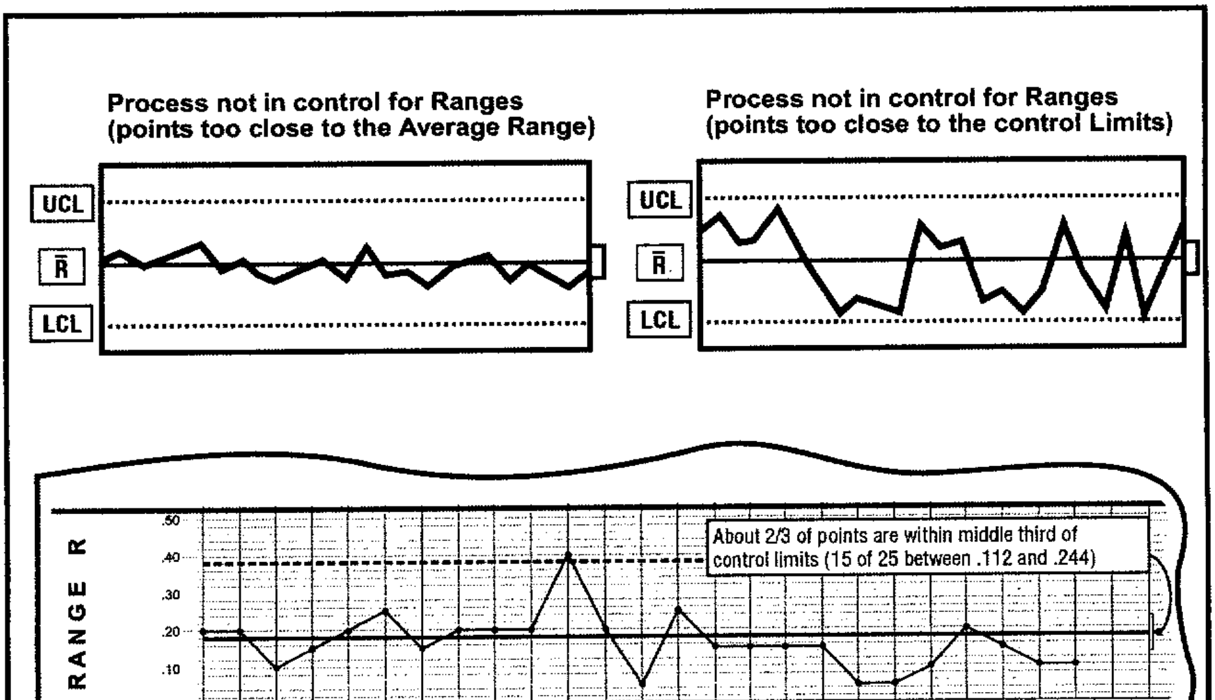
Distance of points from $\bar{R}$ or $\bar{\bar{X}}$: Generally, about 2/3 of the plotted points should lie within the middle third of the region between the control limits; about 1/3 of the points should be in the outer two-thirds of the region. If substantially more than 2/3 of the plotted points lie close to $\bar{R}$ or $\bar{\bar{X}}$ investigate one or more of the following:

- The control limits or plot points have been miscalculated or misplotted.
- The process or the sampling method is stratified; each subgroup systematically contains measurements from two or more process streams that have very different process averages (e.g., one piece from each of several spindles).
- The data have been edited (subgroups with ranges that deviated much from the average have been altered or removed).

If substantially fewer than 2/3 of the plotted points lie close to $\bar{R}$ (for 25 subgroups if 40% or fewer are in the middle third), investigate one or both of the following:

- The control limits or plot points have been miscalculated or misplotted.
- The process or the sampling method causes successive subgroups to contain measurements from two or more process streams that have dramatically different variability (e.g., mixed lots of input materials).

If several process streams are present, they should be identified and tracked separately (see also Appendix A). Figure II.12 shows a nonrandom pattern for the R chart.



SUBGROUP ID													
TIME	8	10	12	2	8	10	12	2	8	10	12	2	8
DATE	6/8				6/9				6/10				6/11
x1	.65	.75	.75	.60	.70	.60	.75	.60	.65	.60	.80	.85	.70
x2	.70	.85	.80	.70	.75	.75	.80	.70	.80	.70	.75	.75	.70
x3	.65	.75	.80	.70	.65	.75	.65	.80	.85	.60	.90	.85	.75
x4	.65	.85	.70	.75	.85	.85	.75	.75	.85	.80	.50	.65	.75
x5	.85	.65	.75	.65	.80	.70	.70	.75	.75	.65	.80	.70	.70
SUM OF INDIVIDUALS	3.50	3.85	3.80	3.40	3.75	3.65	3.65	3.60	3.90	3.35	3.75	3.60	3.60
AVERAGE - $\bar{X}$	.70	.77	.76	.68	.75	.73	.73	.72	.78	.67	.75	.76	.72
RANGE - R	.20	.20	.10	.15	.20	.25	.15	.20	.20	.20	.40	.20	.05

SUBGROUP ID												
TIME	10	12	2	8	10	12	2	8	10	12	2	8
DATE				6/12				6/15				6/16
x1	.65	.90	.75	.76	.75	.65	.60	.50	.60	.80	.65	.65
x2	.70	.80	.80	.70	.70	.65	.60	.55	.80	.65	.60	.70
x3	.85	.80	.75	.85	.60	.65	.65	.65	.65	.75	.65	.70
x4	.75	.75	.80	.70	.70	.65	.60	.60	.85	.65	.60	.60
x5	.60	.85	.65	.80	.60	.70	.65	.60	.75	.65	.70	.65
SUM OF INDIVIDUALS	3.55	4.10	3.75	3.80	3.35	3.30	3.10	3.10	3.45	3.50	3.20	3.30
AVERAGE - $\bar{X}$	.71	.82	.75	.76	.67	.66	.62	.62	.69	.70	.64	.66
RANGE - R	.25	.15	.15	.15	.15	.05	.05	.10	.20	.15	.10	.10

Figure II.12: Nonrandom Patterns in a Control Chart

Special Cause Criteria

There are several criteria for identifying special causes (see table below and AT&T (1984)). The most commonly used are discussed above. The decision as to which criteria to use depends on the process being studied/controlled.

	Summary of Typical Special Cause Criteria
1	1 point more than 3 standard deviations[21] from centerline
2	7 points in a row on same side of centerline
3	6 points in a row, all increasing or all decreasing
4	14 points in a row, alternating up and down
5	2 out of 3 points > 2 standard deviations from centerline (same side)
6	4 out of 5 points > 1 standard deviation from centerline (same side)
7	15 points in a row within 1 standard deviation of centerline (either side)
8	8 points in a row > 1 standard deviation from centerline (either side)

Table II.1

Note 1: Except for the first criterion, the numbers associated with the criteria do not establish an order or priority of use. Determination of which of the additional criteria to use depends on the specific process characteristics and special causes which are dominant within the process.

Note 2: Care should be given not to apply multiple criteria except in those cases where it makes sense. The application of each additional criterion increases the sensitivity of finding a special cause but also increases the chance of a Type I error.

In reviewing the above, it should be noted that not all these considerations for interpretation of control can be applied on the production floor. There is simply too much for the appraiser to remember and utilizing the advantages of a computer is often not feasible on the production floor. So, much of this more detailed analysis may need to be done offline rather than in real time. This supports the need for the process event log and for appropriate thoughtful analysis to be done after the fact.

Another consideration is in the training of operators. Application of the additional control criteria should be used on the production floor when applicable, but not until the operator is ready for it; both with the appropriate training and tools. With time and experience the operator will recognize these patterns in real time.

21 In this table, "standard deviation" refers to the standard deviation used in the calculations of the control limits.

Average Run Length (ARL)

Chapter I stated that decisions made based on charts should balance the risks of Type I errors (over-control, false alarms) to Type II errors (under-control). A measure of this balance is the Average Run Length (ARL).

The Average Run Length is the number of sample subgroups expected between out-of-control signals. The in-control Average Run Length $\left(ARL_0\right)$ is the expected number of subgroup samples between false alarms.

$$ARL_0 = \frac{1}{\Pr\{Type\,I\;Error\}}$$

The ARL is dependent on how out-of-control signals are defined, the true target value's deviation from the estimate, and the true variation relative to the estimate.

Below is a table of approximate $ARL's$ for the standard Shewhart $\bar{X}$ control chart with exceeding the $\pm 3\sigma_{\bar{X}}$ control limits as the only out-of-control signal.

Shift in Target	
$\sigma_{\bar{X}}$	ARL
0	370.4
0.1	352.9
0.2	308.4
0.3	253.1
0.5	155.2
1.0	43.9
1.5	15.0
2.0	6.3
3.0	2.0
4.0	1.2

This table indicates that a mean shift of 1.5 standard deviations (of the mean) would be signaled (on average) by the 15th subgroup after the shift. A shift of 4 standard deviations would be identified within 2 subgroups.

This table also shows that a false signal may be indicated for a process without a shift (i.e., the process remains in statistical control) every 370 subgroups (on average).

Since $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$, the practical magnitude of the shifts can be reduced by increasing the number of items in each subgroup. Larger subgroups reduce the size of $\sigma_{\bar{X}}$ and tighten the control limits around $\bar{\bar{X}}$.

Alternatively, the $ARL's$ can be reduced by adding more out-of-control criteria. Other signals such as runs tests and patterns analysis along with the control limits will reduce the size of the $ARL's$.

The following table is approximate $ARL's$ for the same chart adding the runs test of 7-points in a row on one side of $\bar{\bar{X}}$.

Shift in Target	
$\sigma_{\bar{X}}$'s	*ARL*
0	59.8
0.1	53.9
0.2	41.8
0.3	30.8
0.5	17.9
1.0	8.7
1.5	6.9
2.0	6.1
3.0	2.0
4.0	1.2

As can be seen, adding the one extra out-of-control criterion significantly reduces the *ARLs* for small shifts in the mean, a decrease in the risk of a Type II error. Note that the zero-shift (the in-control) *ARL* is also reduced significantly. This is an increase in the risk of a Type I error or false alarm.

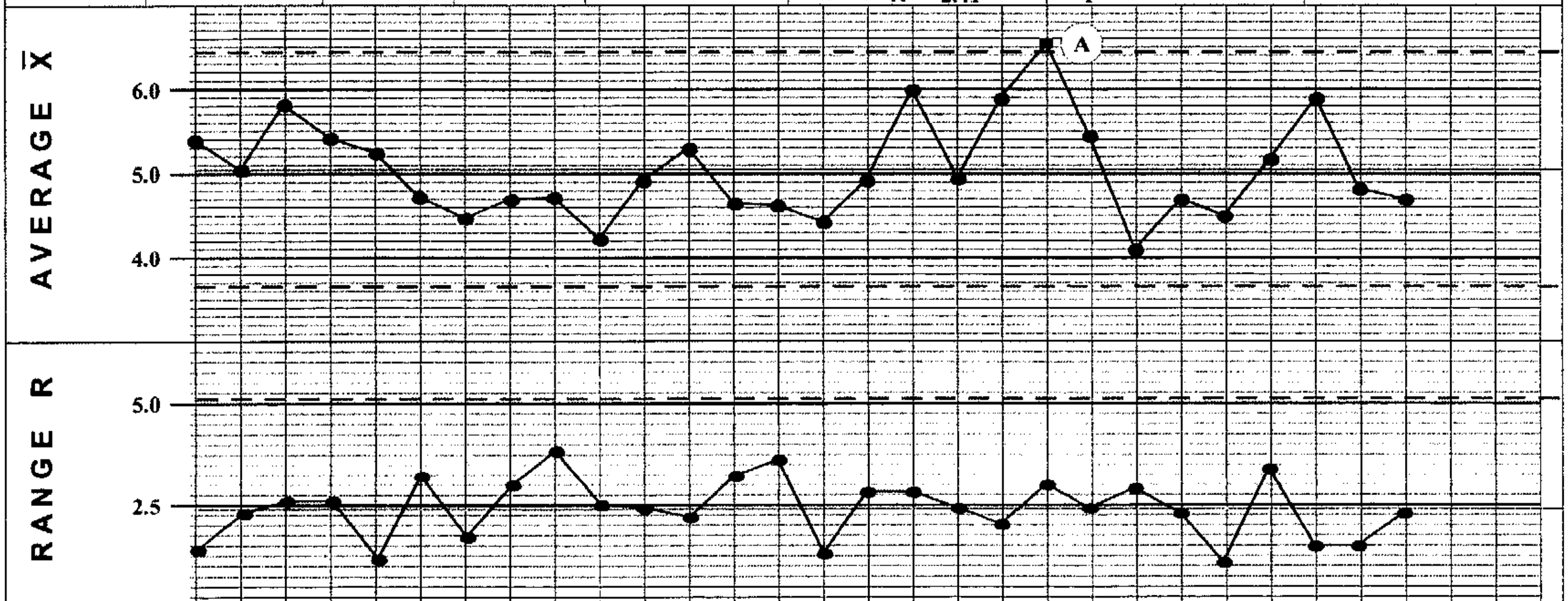
This balance between wanting a long *ARL* when the process is in control versus a short *ARL* when there is a process change has led to the development of other charting methods. Some of those methods are briefly described in Chapter III.

SHEET No. 43

Average and Range Chart

PART NAME	CHARACTERISTIC No.	CHARACTERISTIC	SPECIFICATION	SUBGROUP/SAMPLE SIZE	FREQUENCY	DATE START	DATE STOP
Qubic Release	LD013	DIOUEW Length	5.0 +/- 5.0	5	4 subgps/shift	xx/xx/xx	xx/xx/xx

OPERATION	MACHINE/JIG/FIXTURE No.	MAIN GAGE No.	BACKUP GAGE No.	MEASUREMENT SYSTEM ANALYSIS RESULTS			
VG10	FJG18	PSM05	PSM03	GRR: 13% process	Process Average $\bar{\bar{X}}$ = 5.050 Average Range $\bar{R}$ = 2.41	$UCL_{\bar{X}}$ = 6.438 $LCL_{\bar{X}}$ = 3.662	UCL_R = 5.09 LCL_R = 0.00



SUBGROUP ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14
TIME	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
DATE	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
x1	5.4	4.8	7.2	4.7	4.5	5.7	4.9	6.0	5.9	3.5	3.9	5.6	3.8	5.3
x2	6.2	6.3	4.6	4.3	5.7	4.2	3.7	3.0	3.9	5.7	4.2	4.5	3.2	6.3
x3	5.5	4.0	5.6	6.9	5.3	2.7	4.3	5.3	6.5	3.2	6.2	5.4	4.9	2.7
x4	5.1	6.0	5.1	6.8	5.4	5.1	4.1	3.9	2.7	3.6	6.3	4.4	5.0	4.4
x5	4.8	4.1	6.6	4.4	5.3	5.9	5.4	5.3	4.6	5.2	4.0	6.6	6.4	4.5
SUM OF INDIVIDUALS	27.0	25.2	29.1	27.1	26.2	23.6	22.4	23.5	23.6	21.2	24.6	26.5	23.3	23.2
AVERAGE - X	5.40	5.04	5.82	5.42	5.24	4.72	4.48	4.70	4.72	4.24	4.92	5.30	4.66	4.64
RANGE - R	1.4	2.3	2.6	2.6	1.2	3.2	1.7	3.0	3.8	2.5	2.4	2.2	3.2	3.6

SUBGROUP ID	15	16	17	18	19	20	21	22	23	24	25	26	27	28
TIME	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
DATE	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
x1	3.9	6.5	7.5	6.4	5.6	6.3	4.6	3.5	5.2	4.2	5.2	6.4	4.5	5.4
x2	4.7	4.7	6.4	4.2	6.7	8.1	5.8	5.0	6.1	4.4	5.1	6.4	5.2	5.6
x3	5.2	3.7	4.7	4.6	4.9	5.1	4.7	5.9	3.8	5.0	3.5	5.9	4.9	3.3
x4	4.3	4.2	6.7	5.5	5.4	5.5	5.1	3.0	4.3	5.0	6.9	4.9	4.0	5.4
x5	4.1	5.5	4.7	4.0	6.9	7.7	7.0	3.2	4.1	3.9	5.2	5.9	5.5	3.8
SUM OF INDIVIDUALS	22.2	24.6	30.0	24.7	29.5	32.7	27.2	20.6	23.5	22.5	25.9	29.5	24.1	23.5
AVERAGE - X	4.44	4.92	6.00	4.94	5.90	6.54	5.44	4.12	4.70	4.50	5.18	5.90	4.82	4.70
RANGE - R	1.3	2.8	2.8	2.4	2.0	3.0	2.4	2.9	2.3	1.1	3.4	1.5	1.5	2.3

Figure II.13: Average and Range Charts

CHAPTER II - Section C

Control Chart Formulas

Control chart constants for all control charts discussed in this section are listed in Appendix E.

Variables Control Charts

Average and Range Charts $(\bar{X},\ R)$

Subgroup Average:

$$\bar{X} = \frac{x_1 + x_2 + \ldots + x_n}{n};$$

n = number of samples in a subgroup

Subgroup Range:

$$R = x_{Max} - x_{Min} \quad \text{(within each subgroup)}$$

Grand Average:

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \ldots + \bar{X}_k}{k};$$

k = number of subgroups used to determine the Grand Average and Average Range

Average Range:

$$\bar{R} = \frac{R_1 + R_2 + \ldots . R_k}{k};$$

Estimate of the Standard Deviation of X:

$$\hat{\sigma}_C = \bar{R} / d_2$$

Estimate of the Standard Deviation of $\bar{X}$:

$$\hat{\sigma}_{\bar{X}} = \hat{\sigma}_C / \sqrt{n}$$

Chart Features:

Centerline	Control Limits	
$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$
$CL_R = \bar{R}$	$UCL_R = D_4\bar{R}$	$LCL_R = D_3\bar{R}$

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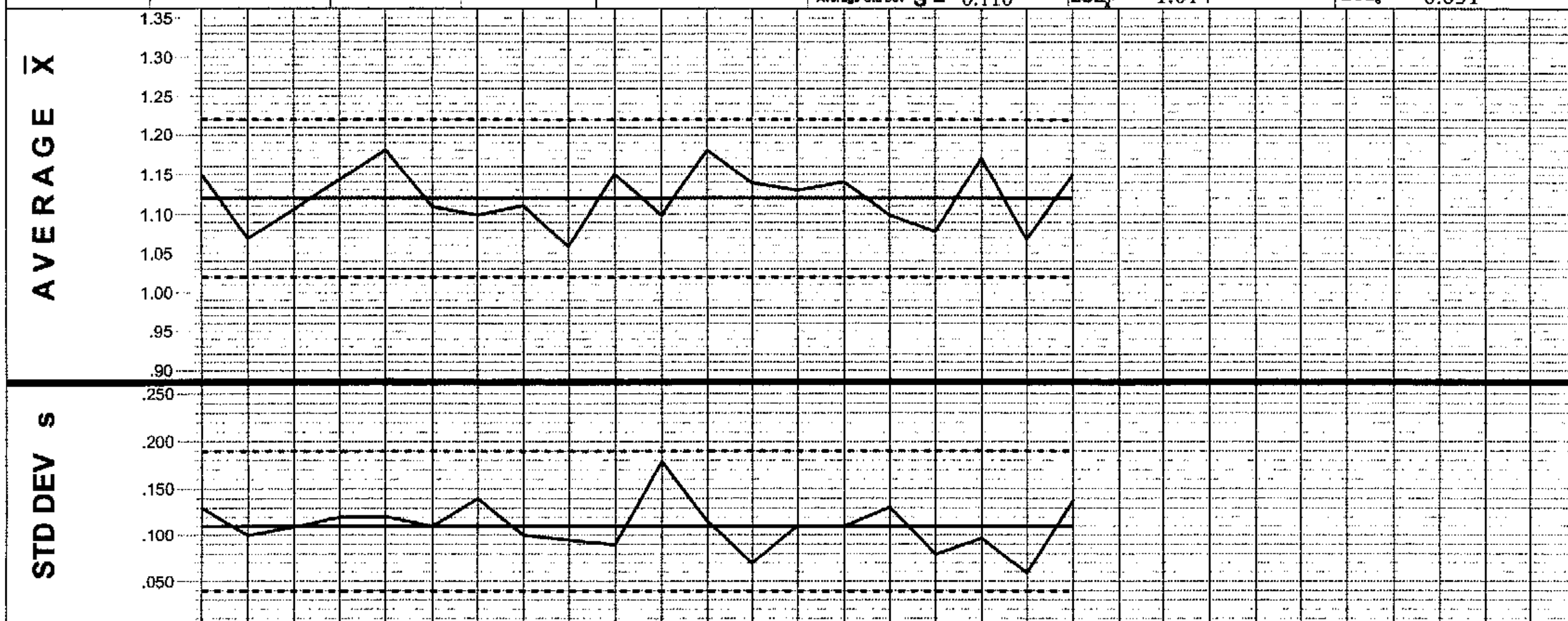
SHEET No.

Average and Standard Deviation Chart

PART NAME	CHARACTERISTIC No.	CHARACTERISTIC	SPECIFICATION	SUBGROUP/SAMPLE SIZE	FREQUENCY	DATE START	DATE STOP
XXX	XXX	XXX	XXX	10	TWICE PER DAY	1-11	1-12

OPERATION	MACHINE/JIG/FIXTURE No.	MAIN GAGE No.	BACKUP GAGE No.	MEASUREMENT SYSTEM ANALYSIS RESULTS			
PRIMER SYSTEM	XXX	XXX	XXX		Process Average $\bar{\bar{X}}$ = 1.122 Average Std Dev $\bar{S}$ = 0.110	$UCL_{\bar{X}}$ = 1.230 $LCL_{\bar{X}}$ = 1.014	UCL_s = 0.189 LCL_s = 0.031



SUBGROUP ID																				
TIME	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
DATE	1/11		1/12		1/13		1/14		1/15		1/18		1/19		1/20		1/21		1/22	
x1																				
x2																				
x3																				
x4																				
x5																				
SUM OF INDIVIDUALS																				
AVERAGE - $\bar{X}$	1.15	1.07	1.10	1.14	1.18	1.11	1.10	1.11	1.06	1.15	1.10	1.19	1.14	1.13	1.14	1.12	1.08	1.18	1.06	1.13
STD DEV - S	.136	.098	.106	.120	.121	.115	.136	.101	.086	.079	.170	.125	.070	.107	.111	.137	.074	.099	.059	.141

Figure II.14: Average and Standard Deviation Charts

Average and Standard Deviation Charts $(\bar{X},\ s)$

Subgroup Average:

$$\bar{X} = \frac{x_1 + x_2 + \ldots + x_n}{n};$$

n = number of samples in a subgroup

Subgroup Standard Deviation (Within-subgroup Variation):

$$s_k = \sqrt{\frac{\sum (X_{i,k} - \bar{X}_k)^2}{n-1}}$$

Grand Average:

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \ldots + \bar{X}_k}{k}$$

k = number of subgroups used to determine the Grand Average and Average Standard Deviation

Average Standard Deviation:[22]

$$\bar{s} = \frac{s_1 + s_2 + \ldots s_k}{k}$$

Estimate of the Standard Deviation of X :

$$\hat{\sigma}_C = \bar{s} / c_4$$

Estimate of the Standard Deviation of $\bar{X}$:

$$\hat{\sigma}_{\bar{X}} = \hat{\sigma}_C / \sqrt{n}$$

Chart Features:

Centerline	Control Limits	
$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_3\bar{s}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{s}$
$CL_s = \bar{s}$	$UCL_s = B_4\bar{s}$	$LCL_s = B_3\bar{s}$

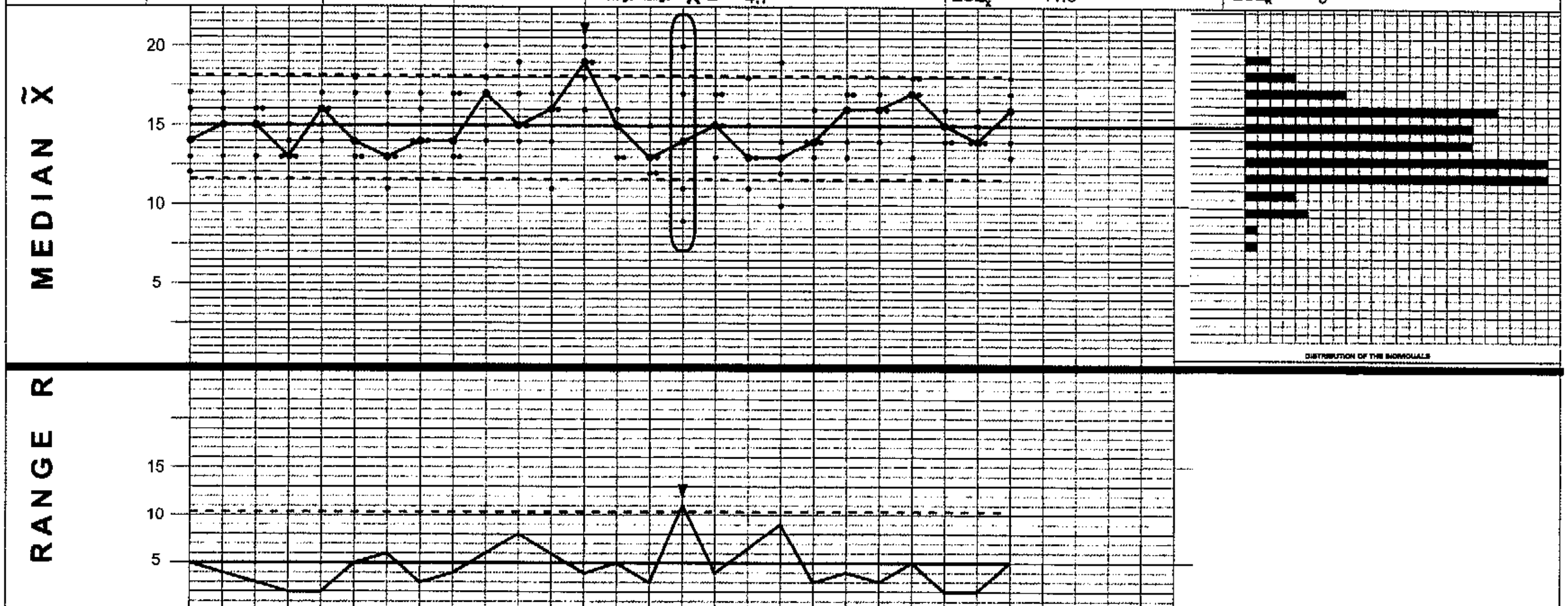
[22] Also known as the pooled standard deviation.

Median and Range Chart

SHEET No.

PART NAME	CHARACTERISTIC No.	CHARACTERISTIC	SPECIFICATION	SUBGROUP/SAMPLE SIZE	FREQUENCY	DATE START	DATE STOP
XXX	XXX	LENGTH (HUNDREDTHS SHOWN ON CHART)	45.65 +/-0.40	5	PER HOUR	2-9	2-10

OPERATION	MACHINE/JIG/FIXTURE No.	MAIN GAGE No.	BACKUP GAGE No.			
CUT-OFF	XXX	XXX	XXX	Process Median $\tilde{\bar{X}} =$ 14.85 Average Range $\bar{R} =$ 4.7	$UCL_{\tilde{X}} =$ 18.1 $LCL_{\tilde{X}} =$ 11.6	$UCL_R =$ 10.3 $LCL_R =$ 0



SUBGROUP ID																														
TIME	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	7	8	9	10	11	12	1	2	3	4					
DATE	2/9																2/10														
x1	16	16	16	15	16	13	13	14	13	15	11	17	20	18	12	11	17	13	19	14	13	16	18	14	15	17					
x2	14	15	13	13	15	13	11	16	17	14	19	16	19	16	13	20	17	15	13	14	17	15	18	15	14	14					
x3	13	13	16	13	16	14	15	17	14	17	14	14	18	13	15	14	15	18	10	16	17	14	17	16	16	18					
x4	12	15	15	13	14	17	17	14	17	20	15	11	19	15	12	17	13	11	12	14	16	17	17	15	14	13					
x5	17	17	15	14	17	18	13	14	13	18	17	16	16	13	13	9	15	13	14	13	14	16	13	14	14	16					
MEDIAN - $\tilde{X}$	14	15	15	13	16	14	13	14	14	17	15	16	19	15	13	14	15	13	13	14	16	16	17	15	14	16					
RANGE - R	5	4	3	2	3	5	6	3	4	6	8	6	4	5	3	11	4	7	9	3	4	3	5	2	2	5					

Figure II.15: Median and Range Charts

Median and Range Charts $\left(\tilde{X}, R\right)$

Sample Value: $x_i, \quad i = 1 \ldots n$ (sample size)

Subgroup Median:

$X^{(O)}$ is the value of the O^{th} element in the sample when the data are arranged in ascending order

$$\tilde{X}_k = \begin{cases} X^{\left(\frac{n+1}{2}\right)} & \text{if } n \text{ is odd} \\ \dfrac{X^{\left(\frac{n}{2}\right)} + X^{\left(\frac{n+2}{2}\right)}}{2} & \text{if } n \text{ is even} \end{cases}$$

$n =$ number of elements in a subgroup

$k =$ number of subgroups used to determine the Average Median and Average Range

Subgroup Range:

$$R = x_{Max} - x_{Min} \quad \text{(within each subgroup)}$$

Average Median:

$$\tilde{\bar{X}} = \frac{\tilde{X}_1 + \tilde{X}_2 + \ldots + \tilde{X}_k}{k};$$

Average Range:

$$\bar{R} = \frac{R_1 + R_2 + \ldots . R_k}{k};$$

Estimate of the Standard Deviation of X:

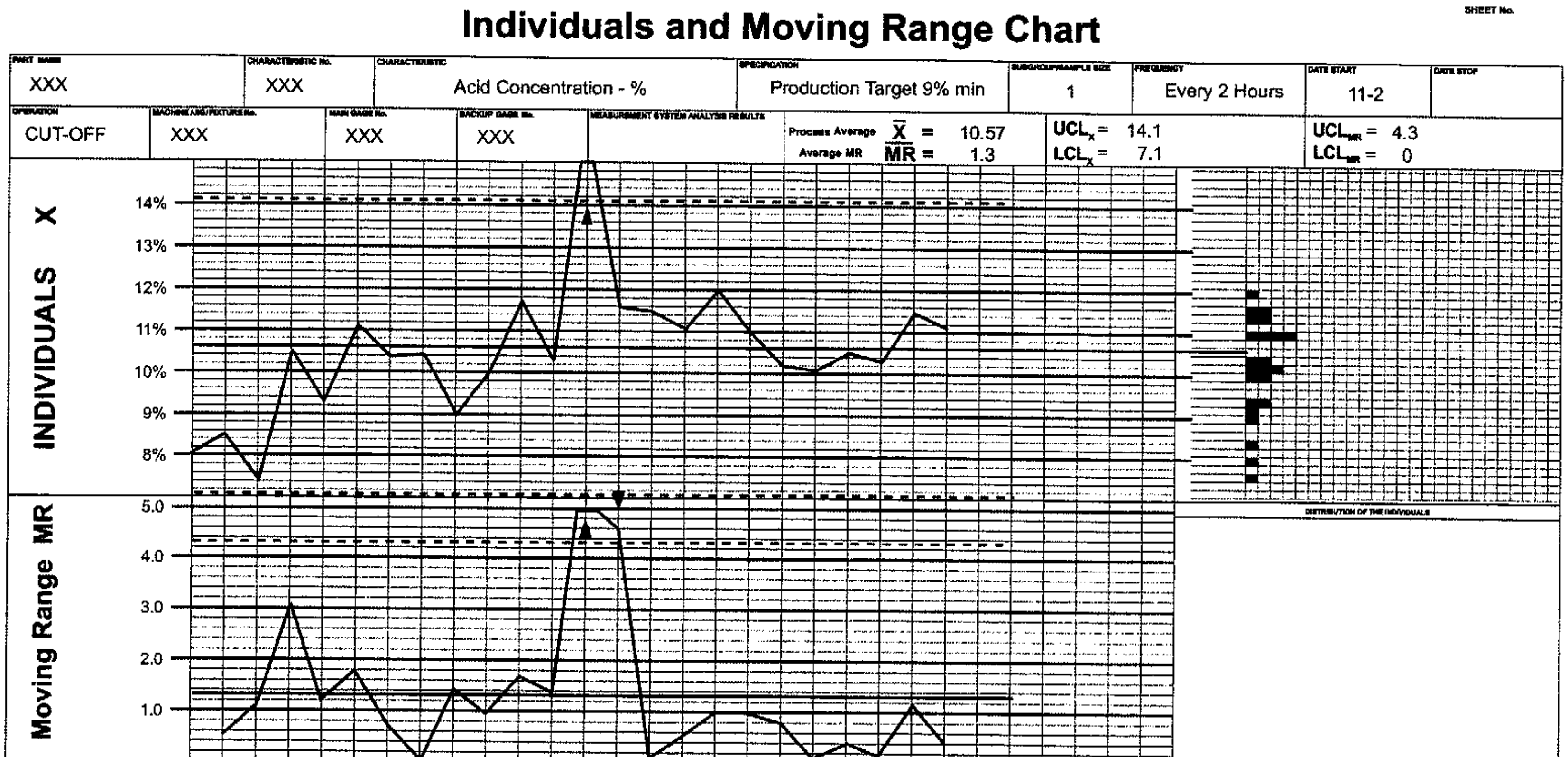
$$\hat{\sigma}_C = \bar{R} / d_2$$

Chart Features:[23]

Centerline	Control Limits	
$CL_{\tilde{X}} = \tilde{\bar{X}}$	$UCL_{\tilde{X}} = \tilde{\bar{X}} + \tilde{\bar{A}}_2 \bar{R}$	$LCL_{\tilde{X}} = \tilde{\bar{X}} - \tilde{\bar{A}}_2 \bar{R}$
$CL_{\bar{R}} = \bar{R}$	$UCL_R = D_4 \bar{R}$	$LCL_R = D_3 \bar{R}$

[23] This approach to the Median Chart uses averages in the calculation of the centerline and control limits. There are other approaches in the literature which do not use averages.

Individuals and Moving Range Chart



SUBGROUP ID																								
TIME	2	2	2	2	3	3	3	3	1	1	1	1	2	2	2	2	3	3	3	3	1	1	1	1
DATE	11/2								11/3												11/4			
VALUE - X	8.0	8.5	7.4	10.5	9.3	11.1	10.4	10.4	9.0	10.0	11.7	10.3	16.2	11.6	11.5	11.0	12.0	11.0	10.2	10.1	10.5	10.3	11.5	11.1
MOVING RANGE - MR		.5	1.1	3.1	1.2	1.8	.7	.0	1.4	1.0	1.7	1.4	5.9	4.6	.1	.5	1.0	1.0	.8	.1	.4	.2	1.2	.4

Figure II.16: Individual and Moving Range Charts

Individuals and Moving Range Charts (X, MR)

Individual Value: $x_i,\ i = 1,\ldots,k$ individual values:

Average of Individual Values:

$$\overline{X} = \frac{x_1 + x_2 + \ldots x_k}{k}$$

Moving Range:

$$MR_i = |x_i - x_{i-1}|,\ i = 2\ldots k$$

(Range between current value and previous value.)

Average Moving Range:

$$\overline{MR} = \frac{MR_2 + MR_3 + \ldots . MR_k}{k-1}$$

Estimate of the Standard Deviation of X:

$$\hat{\sigma}_C = \overline{R} \big/ d_2$$

Chart Features:

Centerline	Control Limits	
$CL_X = \overline{X}$	$UCL_X = \overline{X} + E_2\overline{R}$	$LCL_X = \overline{X} - E_2\overline{R}$
$CL_R = \overline{R}$	$UCL_R = D_4\overline{R}$	$LCL_R = D_3\overline{R}$

Because moving ranges are involved, the points being plotted on the range chart are correlated. Therefore, valid signals occur only in the form of points beyond the control limits. Other rules used to evaluate the data for non-random patterns (see Chapter II, Section B) are not reliable indicators of out-of-control conditions.

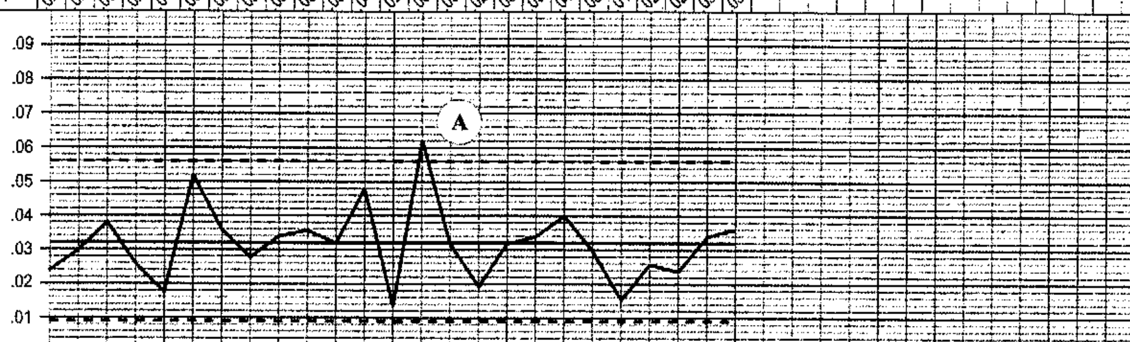
p Chart

SHEET No.

PART NAME	CHARACTERISTIC No.	CHARACTERISTIC	SPECIFICATION	SUBGROUP/SAMPLE SIZE	FREQUENCY	DATE START	DATE STOP
XXX		Functional Requirements		500	Per shift		

OPERATION	MACHINE/JIG/FIXTURE No.	MAIN GAGE No.	BACKUP GAGE No.	MEASUREMENT SYSTEM ANALYSIS RESULTS	Process Average $\bar{p}$ =	UCL_p =	LCL_p =
Final Test		ETS-2730		GRR = 13%	0.0324	0.056	0.009

CHARACTERISTIC	FREQUENCY																									CATEGORY TOTAL	% OF GRAND TOTAL	% OF TOTAL BUILD
Under torque	2	4	8	5	5	9	9	7	8	5	8	7	6	14	6	3	4	8	8	5	5	5	6	9	6			
Non-start	6	4	7	5	2	8	4	3	2	5	4	9	3	8	4	3	3	5	5	5	2	4	4	3	5			
Leaker	4	7	3	3	2	9	4	4	6	8	4	8	2	9	6	4	9	4	6	5	1	4	2	5	6			
Smoker			1				1		1										1						1			
TOTAL UNITS IDENTIFIED (np)	12	15	19	13	9	26	18	14	17	18	16	24	11	31	16	10	16	17	20	15	8	13	12	17	18		GRAND TOTAL	
SAMPLE SIZE (n)	500 →																											
PROPORTION of IDENTIFIED UNITS (p)	.024	.030	.038	.026	.018	.052	.036	.028	.034	.036	.032	.048	.022	.062	.032	.020	.032	.034	.040	.030	.016	.026	.024	.034	.036			



$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

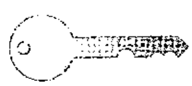
ID No.																									
DATE	May 6	7	8	11	12	13	14	15	18	19	20	21	22	26	27	28	29	Jun 1	2	3	4	5	8	9	10

Figure II.17: Proportion Nonconforming Chart

Attributes Control Charts

Control Charts for Nonconforming Items[24]

Attributes charts are part of probability based charts discussed in Chapter III. These control charts use categorical data and the probabilities related to the categories to identify the presences of special causes. The analysis of categorical data by these charts generally utilizes the binomial, or poisson distribution approximated by the normal form.



Traditionally attributes charts are used to track unacceptable parts by identifying nonconforming items and nonconformities within an item. There is nothing intrinsic in attributes charts that restricts them to be solely used in charting nonconforming items. They can also be used for tracking positive events. However, we will follow tradition and refer to these as nonconformances and nonconformities.

Proportion Nonconforming (*p* Chart)

Guideline:

Since the control limits are based on a normal approximation, the sample size used should be such that $n\bar{p} \geq 5$.

Individual Value

$$p_i = \frac{np_i}{n_i}$$

n_i = number of parts inspected;

np_i = number of nonconforming items found

Average of Individual Values

$$\bar{p} = \frac{np_1 + np_2 + \ldots. + np_k}{n_1 + n_2 + \ldots. + n_k}$$

where k = number of subgroups

$$\bar{p} = \frac{p_1 + p_2 + \ldots. + p_k}{k}$$

if all the n_i's are equal

[24] An alternative to these charts is the Individuals and Moving Range Chart (see Wheeler (1995)).

Chart Features:

Centerline

$$CL_p = \bar{p}$$

Control Limits

$$UCL_{p_i} = \bar{p} + 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n_i}}$$

$$LCL_{p_i} = \bar{p} - 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n_i}}$$

If the sample size is constant (*n*)

Control Limits

$$UCL_p = \bar{p} + 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$$

$$LCL_p = \bar{p} - 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$$

Constant control limits when the sample size varies

(for situations where $\frac{\min n_i}{\max n_i} \geq 0.75$)

Control Limits

$$UCL_p = \bar{p} + 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{\bar{n}}} \qquad (\bar{n} = \text{average sample size})$$

$$LCL_p = \bar{p} - 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{\bar{n}}} \qquad (\bar{n} = \text{average sample size})$$

Example Uses:

- Accept/Reject Decisions with **constant or variable** subgroup size
  - ✓ First Time Quality (FTQ) results[25]
  - ✓ Proportion nonconforming
  - ✓ Proportion conforming[26]
  - ✓ Proportion of items above (or below) a threshold value
- Judgment Decisions
  - ✓ Proportion of items within a specified category
  - ✓ Proportion of items above (or below) a threshold value
  - ✓ Proportion Uptime (equipment)

25 This is alternatively known as FTC (First Time Capability) and RTY (Rolled Throughput Yield).
26 This chart is sometimes called a q-chart; this is based on the practice of calculating the parameter $q = 1 - p$.

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np Chart

SHEET No.

PART NAME	CHARACTERISTIC No.	CHARACTERISTIC	SPECIFICATION	SUBGROUP/SAMPLE SIZE	FREQUENCY	DATE START	DATE STOP
XXX		Spot Welds - flaws		62 welds/ housing	1 housing/half hr.		

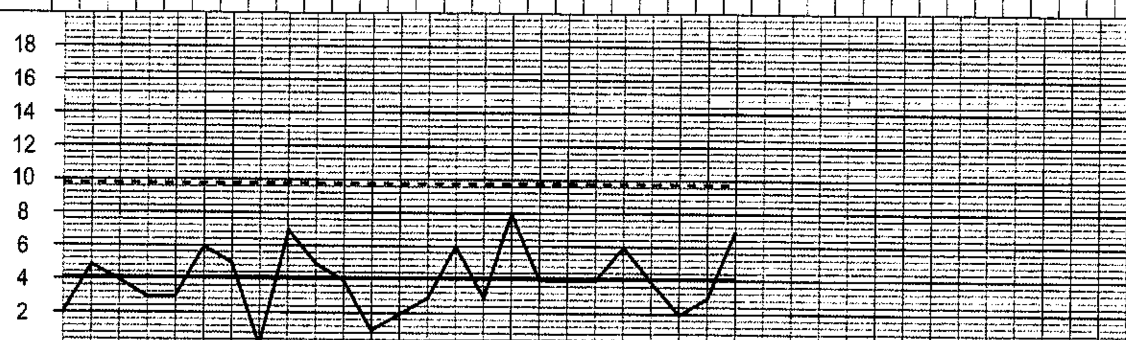
OPERATION	MACHINE/JIG/FIXTURE No.	MAIN GAGE No.	BACKUP GAGE No.	MEASUREMENT SYSTEM ANALYSIS RESULTS	Process Average	UCL	LCL
XXX		VS-5615		Acceptable	$\overline{np}$ =4.04	UCL_{np} = 9.87	LCL_{np} = —

CHARACTERISTIC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Undersize	2	4	3	3	2	6	4	0	6	5	4	1	2	2	6	3	8	4	3	4	5	4	2	3	6
Cold weld							1												1						
Missing		1			1									1							1				
Off-location			1						1																1
TOTAL UNITS IDENTIFIED (*np*)	2	5	4	3	3	6	5	0	7	5	4	1	2	3	6	3	8	4	4	4	6	4	2	3	7
SAMPLE SIZE (*n*)	62 →																								

FREQUENCY

CATEGORY TOTAL	% OF GRAND TOTAL	% OF TOTAL BUILD

GRAND TOTAL



ID No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
DATE March	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2

$$UCL = \overline{np} + 3\sqrt{\overline{np}\,(1-\overline{p})}$$

$$LCL = \overline{np} - 3\sqrt{\overline{np}\,(1-\overline{p})}$$

Figure II.18: Number of Nonconforming Chart

Number of Nonconforming Chart (*np* Chart)

Restriction:

Requires a constant subgroup size = n

Guideline:

Since the control limits are based on a normal approximation, the sample size used should be such that $\overline{np} \geq 5$

Individual Value:

np_i $\quad$ n = number of parts inspected;
np = number of nonconforming items found

Average of Individual Values:

$$\overline{np} = \frac{np_1 + np_2 + ... + np_k}{k}$$

Chart Features:

Centerline $\quad$ **Control Limits:**

$$CL_{np} = \overline{np} \qquad UCL_{np} = \overline{np} + 3\sqrt{\overline{np}(1 - \frac{\overline{np}}{n})} = \overline{np} + 3\sqrt{\overline{np}(1 - \overline{p})}$$

$$LCL_{np} = \overline{np} - 3\sqrt{\overline{np}(1 - \frac{\overline{np}}{n})} = \overline{np} - 3\sqrt{\overline{np}(1 - \overline{p})}$$

Example Uses:

- Accept/Reject Decisions with **constant** subgroup size
  - ✓ First Time Quality (FTQ) results
  - ✓ Number nonconforming
  - ✓ Number conforming
  - ✓ Number of items above (or below) a threshold value
- Judgment Decisions
  - ✓ Number of items within a specified category
  - ✓ Number of items above (or below) a threshold value
  - ✓ Number of times a condition occurs

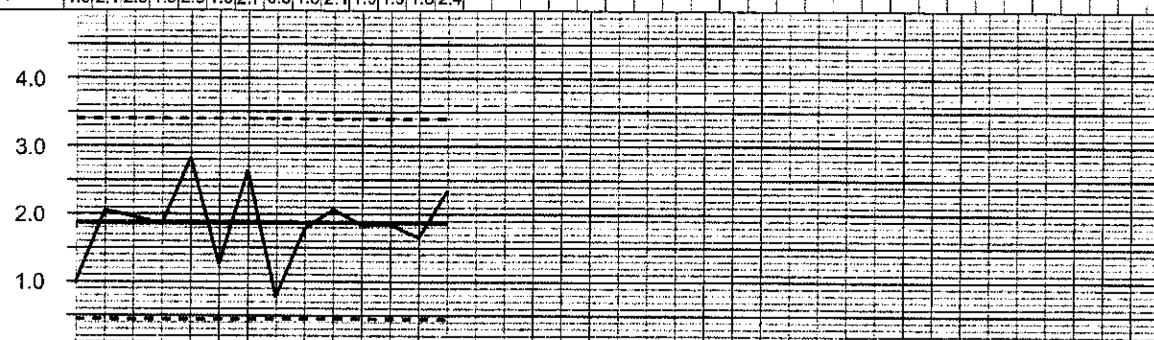
u Chart

SHEET No.

PART NAME	CHARACTERISTIC No.	CHARACTERISTIC	SPECIFICATION	SUBGROUP/SAMPLE SIZE	FREQUENCY	DATE START	DATE STOP
XXX		All Nonconformances		8 boxes	Per day		

OPERATION	MACHINE/JIG/FIXTURE No.	MAIN GAGE No.	BACKUP GAGE No.	MEASUREMENT SYSTEM ANALYSIS RESULTS	Process Average	UCL	LCL
Dock Audit		VS-845		Acceptable	$\bar{u}$ = 1.89	UCL_u = 3.35	LCL_u = 0.43

CHARACTERISTIC	FREQUENCY														CATEGORY TOTAL	% OF GRAND TOTAL	% OF TOTAL BUILD
nd013	1	1					3		1	3	2		6				
nd018	1	1	1		3		7		1	1	1	1		6			
nd043		3	7		1		2		2	2	1			9			
nd050	2	1			1		1		1	1	1	1		2			
nd084	3	3	2		2		1	3	1	1	1	1		1			
vr721		4	1		1	6	1	1	1		3	1		1			
vr1050		1	3	4		1	1	2	1	1	1	1		1			
vr1901	1	1			2	1	2		3	3	3	4		3			
vr1942			4	4	5		1		1	1		1	5				
vr3017		2		7	8	1			2	4		5	5				
TOTAL ITEMS IDENTIFIED (u)	8	17	18	15	23	9	19	6	14	17	13	15	16	22	GRAND TOTAL		
SAMPLE SIZE (n)	8	8	9	8	8	7	7	8	8	8	7	8	9	9			
ITEMS PER UNITS (u/n)	1.0	2.1	2.0	1.9	2.9	1.3	2.7	0.8	1.8	2.1	1.9	1.9	1.8	2.4			



ID No.															
DATE	Feb	9	10	11	12	15	16	17	18	19	22	23	24	25	26

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

Figure II.19: Number of Nonconforming per Unit Chart

Number of Nonconformities per Unit Chart (*u* Chart)

Guideline:

Since the control limits are based on a normal approximation, the sample size used must be large enough so that the number of subgroups with $c = 0$ is small.

Individual Value:

$u_i = \frac{c_i}{n_i}$ c_i = number of nonconformities found in sample i;

n_i = is the sample size

Average of Individual Values:

$$\bar{u} = \frac{u_1 + u_2 + ... + u_k}{k}$$

Chart Features:

Centerline

$$CL_u = \bar{u}$$

Control Limits

$$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{n_i}} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$$

$$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{n_i}} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$$

For constant control limits when the sample size varies

(for situations where $\frac{\min\ n_i}{\max\ n_i} \geq 0.75$)

Control Limits:

$$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{\bar{n}}} = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}} \qquad (\bar{n} = \text{average sample size})$$

$$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{\bar{n}}} = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}} \qquad (\bar{n} = \text{average sample size})$$

Example Uses:

- Accept/Reject Decisions with **variable** number items per unit
  - ✓ Quality rates for specified unit designation
  - ✓ Average number (rate) of nonconformities per unit
  - ✓ Average number (rate) of items within one or more categories
- Judgment Decisions
  - ✓ Average number (rate) of items within one or more categories
  - ✓ Average number (rate) of items above (or below) a threshold value per unit

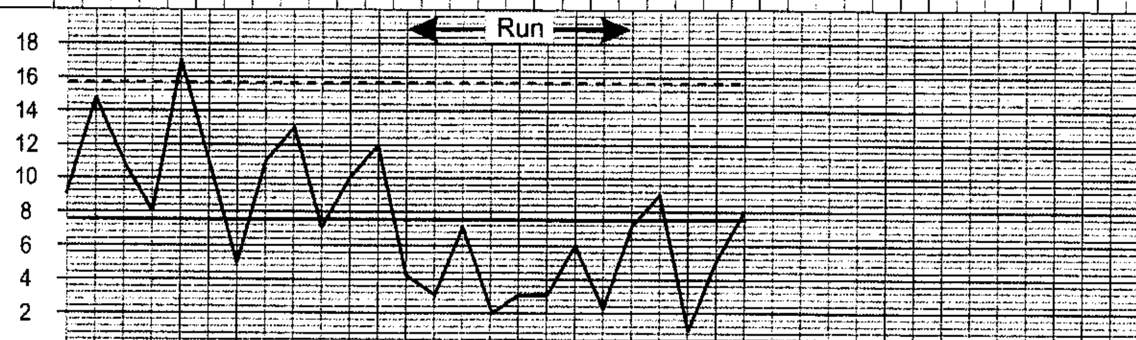
c Chart

SHEET No.

PART NAME	CHARACTERISTIC No.	CHARACTERISTIC	SPECIFICATION	SUBGROUP/SAMPLE SIZE	FREQUENCY	DATE START	DATE STOP
XXX		All Flaws		Whole bolt	Each bolt		

OPERATION	MACHINE/JIG/FIXTURE No.	MAIN GAGE No.	BACKUP GAGE No.	MEASUREMENT SYSTEM ANALYSIS RESULTS	Process Average	UCL	LCL
		VS-1041		Acceptable	$\bar{c}$ = 7.56	UCL_c = 15.81	LCL_c = --

CHARACTERISTIC	FREQUENCY																									CATEGORY TOTAL	% OF GRAND TOTAL	% OF TOTAL BUILD
vr721	4	1			6	4		4	4	3		1	1	4	1		1		1					1				
vr1050	1	3			1	1	2	1	1	1		1		1	3	2		1	1	2	1	1	1					
vr1901	2	7			1	2		3	3	3	4		1	1			2	1	2		3	3		4				
vr1942		4	4		8	1		1	1		1	5	2		4				3		1	1			3			
vr3017	2		7	8	1	3	3	2	4		5	5		2				1			2	4			5			
TOTAL ITEMS IDENTIFIED (c)	9	15	11	8	17	11	5	11	13	7	10	12	4	3	7	2	3	3	6	2	7	9	1	5	8		GRAND TOTAL	
SAMPLE SIZE (n)	1 →																											



ID No.	1	2	3	4	5	1	2	3	1	2	3	4	5	1	2	3	4	1	2	3	4	5	1	2	3
DATE	11 9	11 10	10	10	10	10	10	11 16	16	16	16	16	16	11 17	17	17	17	11 18	18	18	18	18	11 19	19	19

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

Figure II.20: Number of Nonconformities Chart

Number of Nonconformities Chart (c Chart)

Restriction:

Requires a constant subgroup size = n

Guideline:

Since the control limits are based on a normal approximation, the sample used must be large enough so that the number of subgroups with $c = 0$ is small

Individual Value:

c_i = number of nonconformities found in sample; $i = 1,\ldots,k$

Average of Individual Values:

$$\bar{c} = \frac{c_1 + c_2 + \ldots. + c_k}{k} \quad k = \text{number of samples}$$

Chart Features:

Centerline

$$CL_C = \bar{c}$$

Control Limits

$$UCL_C = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_C = \bar{c} - 3\sqrt{\bar{c}}$$

Example Uses:

- Accept/Reject Decisions with a **constant** number items per unit
  - ✓ Quality level for specified unit designation
  - ✓ Total number of nonconformities per unit
  - ✓ Total number of items within one or more categories
- Judgment Decisions
  - ✓ Total number of items within one or more categories per unit
  - ✓ Total number of items above (or below) a threshold value per unit
  - ✓ Total number of times a condition occurs within a unit

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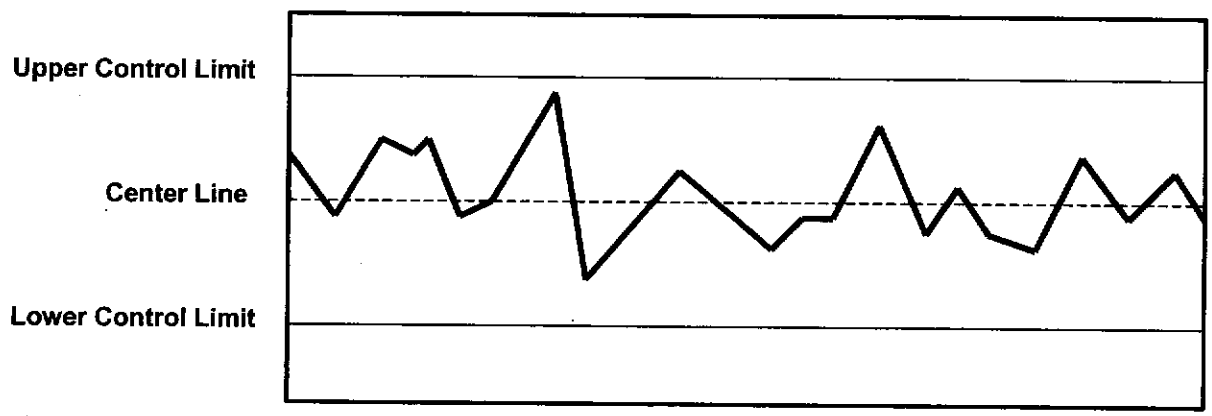
CHAPTER III

Other Types of Control Charts

CONTROL CHARTS



1. **Collection**
   - Gather Data and plot on a chart.
2. **Control**
   - Calculate trial control limits from process data.
   - Identify special causes of variation and act upon them.
3. **Analysis and Improvement**
   - Quantify common cause variation; take action to reduce it.

These three phases are repeated for continual process improvement

Figure III.1: Control Charts

Introduction

There are several types of control charts other than those discussed in the previous chapters. Most of these charts were developed to address specific process situations or conditions which can affect the optimal use of the standard control charts. A brief description of the more common charts will follow below. This description will define the charts, discuss when they should be used and list the formulas associated with the chart, as appropriate. If more information is desired regarding these charts or others, please consult a reference text that deals specifically with these types of control charts.

Probability Based Charts

Probability based charts belong to a class of control charts that uses categorical data and the probabilities related to the categories. The analysis of categorical data generally uses the binomial, multinomial or poisson distribution. Examples of these charts are the attributes charts discussed in Chapter II Section C. The attributes charts use the categories of "good" and "bad" (e.g., conforming and nonconforming). However, there is nothing inherent in any of these forms (or any other forms) that requires one or more categories to be "bad."

The problem is that users tend to apply by example, rather than by knowledge. This is as much the fault of professionals and teachers, as it is the student's. There is a tendency to take the easy way out, using traditional (and stereotypical) examples. This leads to a failure to realize that quality practitioners once had (or were constrained to) the tolerance philosophy; i.e., make it "to print" (or "close enough").

Stoplight Control

With stoplight control charts, the process location and variation are controlled using one chart. The chart tracks the number of data points in the sample in each of the designated categories. The decision criteria are based on the expected probabilities for these categories.

A typical scenario will divide the process variation into three parts: warning low, target, warning high. The areas outside the expected process variation ($6\hat{\sigma}$) are the stop zones. One simple but effective control procedure of this type is stoplight control which is a semi-variables (more than two categories) technique using double sampling. In this approach the target area is designated green, the warning areas as yellow, and the stop zones as red. The use of these colors gives rise to the "stoplight" designation.

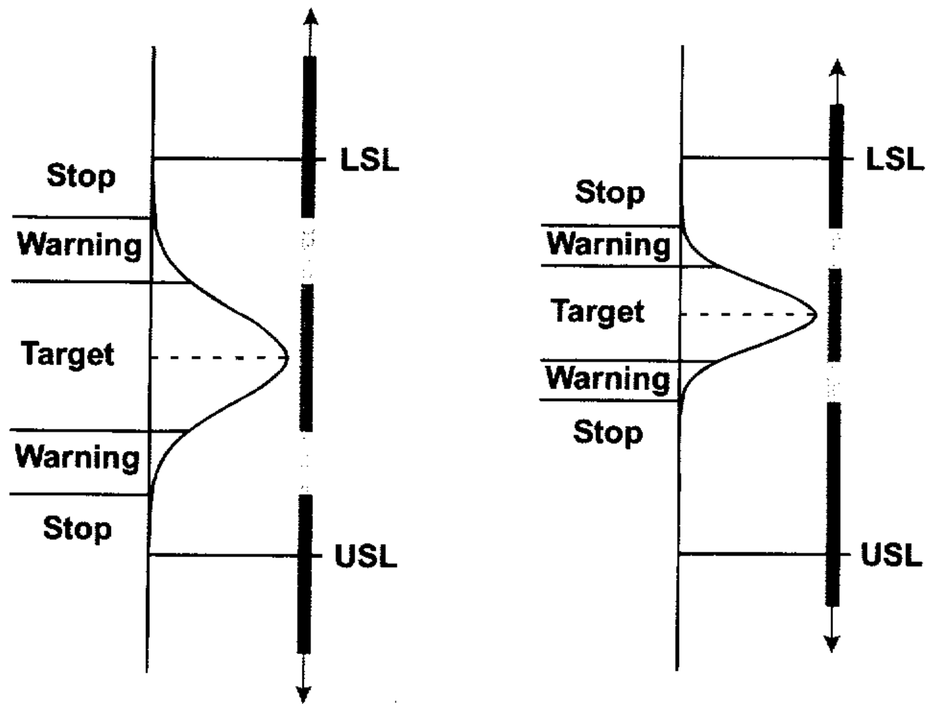


Figure III.2: Stoplight Control

With this categorization the process can be controlled by identifying and charting the proportion of data points designated as "warning" within a sample. The apportionment (% warning) controls the sample size and frequency required. Of course, this allows process control only if the process distribution is known. The quantification and analysis of the process requires variables data.

The focus of this tool is to detect changes (special causes of variation) in the process. That is, this is an appropriate tool for stage 2 activities[27] only. At its basic implementation, stoplight control requires no computations and no plotting, thereby making it easier to implement than control charts. Since it splits the total sample (e.g., 5) into a two-stage sampling (e.g., 2, 3), this approach can signal out-of-control conditions with the same or better efficiency than a control chart with the same total sample size (see Heaphy and Gruska (1982)).

Although, the development of this technique is thoroughly founded in statistical theory, it can be implemented and taught at the operator level without involving mathematics.

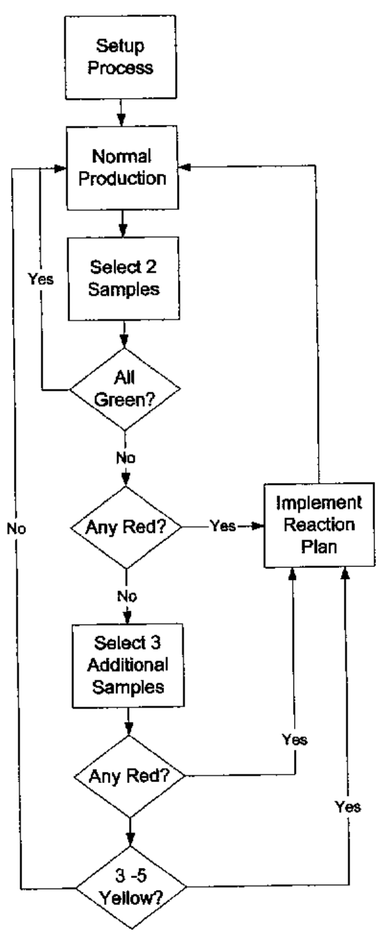
[27] See Chapter I, Section F.

The assumptions in stoplight control are:

- The process is in statistical control.
- Process performance (including measurement variability) is acceptable.
- The process is on target.

Once the assumptions have been verified by a process performance study using variables data techniques, the process distribution can be divided such that the average $\pm$ 1.5 standard deviations is labeled as the green area and the rest of the area within the process distribution is yellow. Any area outside the process distribution (the 99.73% range) is labeled red.

If the process distribution follows the normal form, approximately 86.6% of the distribution is in the green area, 13.2% is in the yellow area and 0.3% is in the red area. Similar conditions can be established if the distribution is found to be non-normal.



For control equivalent to an $\bar{X}$ *and* R chart with a sample size of 5, the steps for stoplight control can be outlined as follows:

1. Check 2 pieces; if both pieces are in the green area, continue to run.
2. If one or both are in the red zone, stop the process, notify the designated person for corrective action and sort material. When setup or other corrections are made, repeat step # 1.
3. If one or both are in a yellow zone, check three more pieces. If any pieces fall in a red zone, stop the process, notify the designated person for corrective action and sort material. When setup or other corrections are made, repeat step # 1.
   - ✓ If no pieces fall in a red zone, but three or more are in a yellow zone (out of 5 pieces) stop the process, notify the designated person for corrective action. When setup or other corrections are made, repeat step #1.
   - ✓ If three pieces fall in the green zone and the rest are yellow, continue to run.

Measurements can be made with variables as well as attributes gaging. Certain variables gaging such as dial indicators or air-electronic columns are better suited for this type of program since the indicator background can be color coded. Although no charts or graphs are required, charting is recommended, especially if subtle trends (shifts over a relatively long period of time) are possible in the process.

In any decision-making situation there is a risk of making a wrong decision. With sampling, the two types of errors are:

- Probability of calling the process bad when it is actually good (false alarm rate).
- Probability of calling the process good when it is actually bad (miss rate).

In addition to these two measures, the sensitivity of the sampling plan can be quantified. Sensitivity refers to the ability of the sampling plan to detect out-of-control conditions due to increased variation or shifts from the process average.

The disadvantage of stoplight control is that it has a higher false alarm rate than an $\bar{X}$ *and* R chart of the same total sample size.

The advantage of stoplight control is that it is as sensitive as an $\bar{X}$ *and* R chart of the same total sample size.

Users tend to accept control mechanisms based on these types of data due to the ease of data collection and analysis. Focus is on the target not specification limits – thus it is compatible with the target philosophy and continuous improvement.

Pre-Control

An application of the stoplight control approach for the ***purpose of nonconformance control instead of process control*** is called Pre-control. It is based on the specifications not the process variation. Its origins can be traced to work by Frank Satterthwaite from Rath & Strong at the Jones & Lamson Machine Company in 1954.[28]

The assumptions in pre-control are:

- the process has a flat loss function (see section on Loss Function, in Chapter IV.)
- process performance (including measurement system variability) is less than or equal to the tolerance.

The first assumption means that all special sources of variation in the process are being controlled. The second assumption states that 99.73% of the pieces being produced are within specification without sorting.

If the foregoing assumptions are satisfied, the tolerance can be divided so that Nominal ± ¼ Tolerance is labeled as the green area and the rest of the area within the specification is yellow. The area outside the specifications is labeled red. For a process that is normal with C_p, C_{pk} equal to 1.00, approximately 86.6% of the pieces are in the green area, 13.2% are in the yellow area and 0.3% are in the red area. Similar calculations could be done if the distribution was found to be non-normal or highly capable.

The pre-control sampling uses a sample size of two. However, before the sampling can start, the process must produce 5 consecutive parts in the green zone. Each of the two data points are plotted on the chart and reviewed against a set of rules.

[28] See Bhote (1991) and ASQ Statistics Newsletter Vol 05 No 2 Feb. 1984.

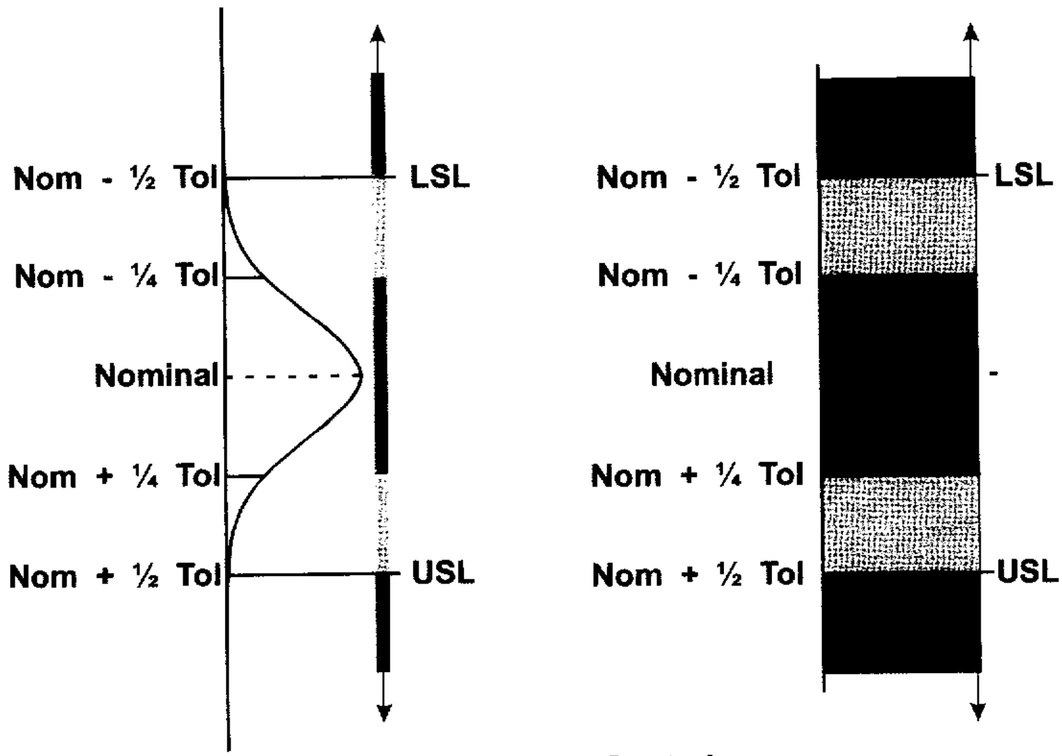


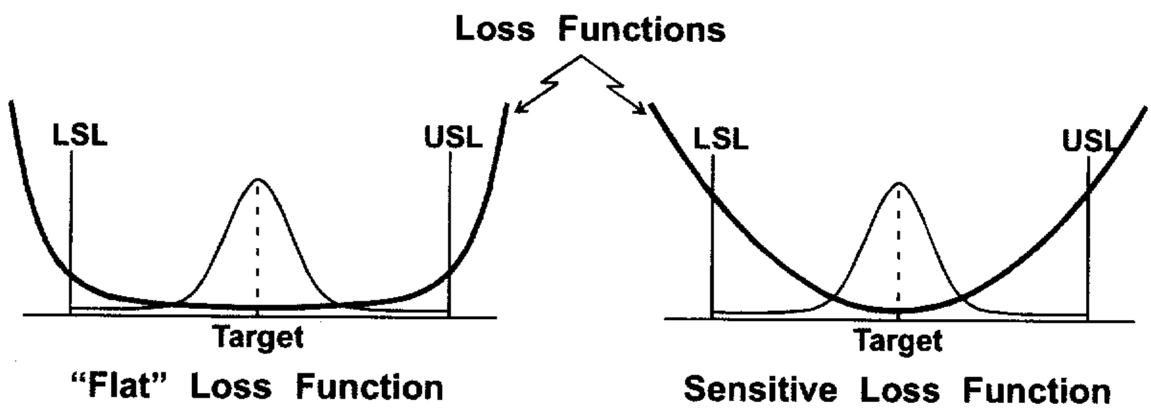
Figure III.3: Pre-Control

When using a pre-control the following rules should be used.

- Two data points in the green zone – continue to run the process.
- One data point in the green zone and one data point in the yellow zone – continue to run the process.
- Two yellow points in a row (same zone) – adjust the process
- Two yellow points in a row (opposite zone) – stop the process and investigate
- One red data point – stop the process and investigate.

Every time the process is adjusted, before the sampling can start the process must produce 5 consecutive parts in the green zone.

Pre-control is not a process control chart but a nonconformance control chart so great care must be taken as to how this chart is used and interpreted. Pre-control charts should be not used when you have a C_p, C_{pk} greater than one or a loss function that is not flat within the specifications (see Chapter IV).



The benefit of pre-control is its simplicity. The disadvantage of pre-control is that potential diagnostics that are available with normal process control methods are not available. Further, pre-control does not evaluate nor monitor process stability. Pre-control is a compliance based tool not a process control tool.

Short-Run Control Charts

Standard control chart approaches are well suited for long production runs. However there are processes that only produce a small number of products during a single run (e.g., job shops). Further, the increasing focus on just-in-time (JIT) inventory and lean manufacturing methods is driving production runs to become shorter. From a business perspective, producing large batches of product several times per month and holding it in inventory for later distribution, can lead to avoidable, unnecessary costs. Manufacturers now are moving toward JIT – producing much smaller quantities on a more frequent basis to avoid the costs of holding "work in process" and inventory. For example, in the past, it may have been satisfactory to make 10,000 parts per month in batches of 2,500 per week. Now, customer demand, flexible manufacturing methods and JIT requirements might lead to making and shipping only 500 parts per day.

To realize the efficiencies of short-run processes it is essential that SPC methods be able to verify that the process is truly in statistical control, (i.e., predictable), and be able to detect special-cause variation during these "short runs."

Wheeler (1991) describes four requirements for an "Ideal State" of process operation essential for competing in this arena:

a. "The process must be inherently stable over time.

b. The process must be operated in a stable and consistent manner.

c. The process aim must be set and maintained at the proper level.

d. The Natural Process Limits must fall within the specification limits."

Effective control charts can be constructed even with small amounts of data. Short-run oriented charts allow a single chart to be used for the control of multiple products. There are a number of variations on this theme. Among the more widely described short-run charts are: [29]

a. **Difference or Deviation from Nominal (DNOM) $\bar{X}$ & R chart.** Production processes for short runs of different products can be characterized easily on a single chart by plotting the differences between the product measurement and its target value. These charts can be applied both to individual measurements and to grouped data.

[29] Caution should be used when subgroups are formed from small populations or when the subgroups use measurements taken over extended periods of time (see Appendix A). Wheeler (1991) discusses evaluating the data with an Individuals and Moving Range (*I & MR*) chart to ensure that important process behavior information is not being masked by the subgrouping.

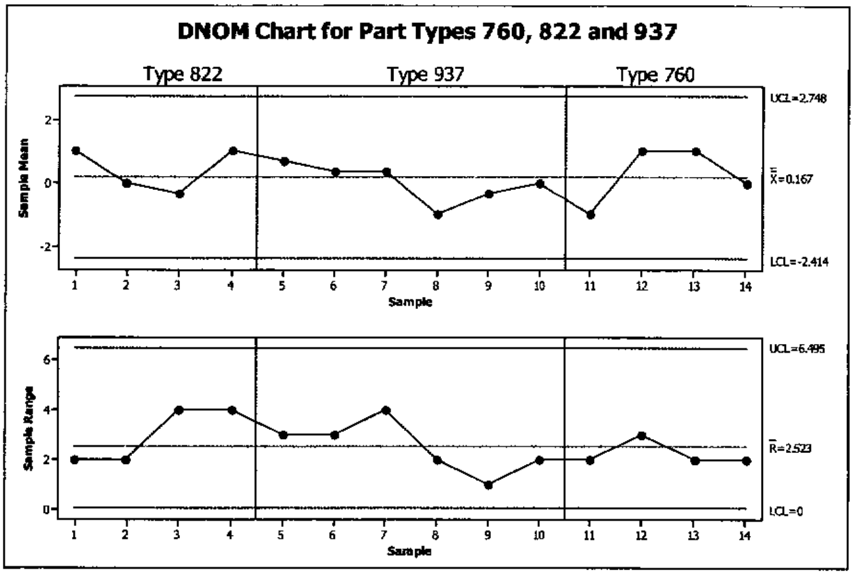


Figure III.4: DNOM Control Chart

b. Standardized $\overline{X}$ & R chart.

The DNOM approach assumes a common, constant variance among the products being tracked on a single chart. When there are substantial differences in the variances of these products, using the deviation from the process target becomes problematic. In such cases the data may be standardized to compensate for the different product means and variability using a transformation of the form:

$$Z = \frac{X - \mu}{\sigma}$$

This class of charts sometimes is referred to as Z or *Zed* charts.

In some short-run processes, the total production volume may be too small to utilize subgrouping effectively. In these cases subgrouping measurements may work counter to the concept of controlling the process and reduce the control chart to a report card function. But when subgrouping is possible, the measurements can be standardized to accommodate this case.

c. Standardized Attributes Control Charts.

Attributes data samples, including those of variable size, can be standardized so that multiple part types can be plotted on a single chart. The standardized statistic has the form:

$$Z_i = \frac{\text{Difference from Mean}}{\text{Standard Deviation}}$$

For example, a u statistic for defect rate would be standardized as:

$$Z_i = \frac{u_i - \bar{u}}{\sqrt{\bar{u}/n}}$$

This method also applies to *np, p, c* and *u* charts.

See Farnum (1992), Juran and Godfrey (1999), Montgomery (1997), Wheeler (1991) and Wise and Fair (1998) for detailed discussions and examples of short-run applications.

Charts for Detecting Small Changes

There are situations where small changes in the process mean can cause problems. Shewhart control charts may not be sensitive enough to efficiently detect these changes, e.g., less than 1.5σ. The two alternative charts discussed here were developed to improve sensitivity for detecting small excursions in the process mean. While the typical Shewhart chart uses only the information supplied by the most recent datum point, the Cumulative Sum (CUSUM) and the Exponentially Weighted Moving-Average (EWMA) charts exploit the information available in accumulated, historical data. See Montgomery (1997), Wheeler (1995) and Grant and Leavenworth (1996) for in-depth discussions of these methods and comparisons with the supplemental detection rules for enhancing the sensitivity of the Shewhart chart to small process shifts

CUSUM (Cumulative Sum) Chart

A CUSUM chart plots the cumulative sum of deviations of successive sample means from a target specification so that even minor permanent shifts (0.5 sigma or below) in the process mean will eventually signal that a shift has occurred. For larger shifts, Shewhart control charts are just as effective and take less effort.

These charts are most often used to monitor continuous processes, such as in the chemical industry, where small shifts can have significant effects.

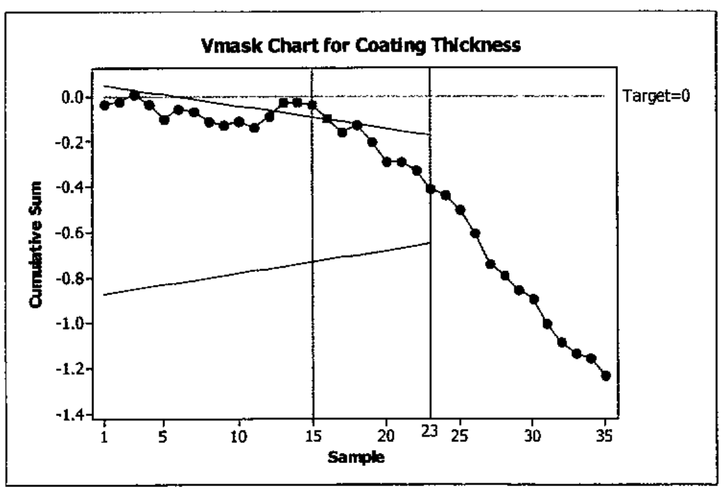
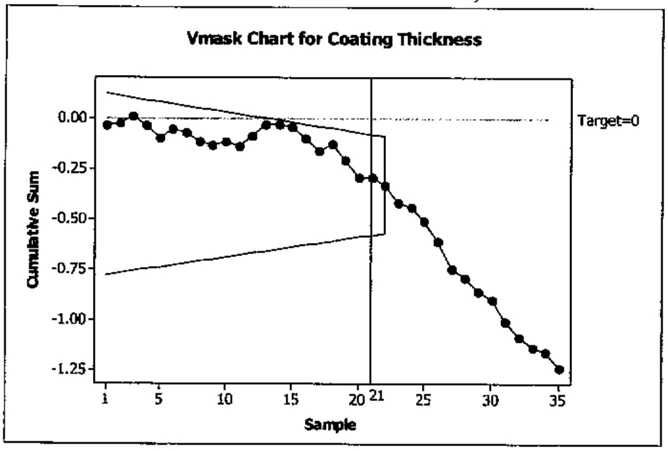


Figure III.5: CUSUM Chart with V-Mask

The CUSUM chart evaluates the slope of the plotted line. A graphical tool (V-mask) is laid over the chart with a vertical reference line offset from origin of the V passing through the last plotted point (see Figure III.5). The offset and angle of the arms are functions of the desired level of sensitivity to process shifts. An out-of-control condition (e.g., a significant process shift) is indicated when previously plotted points fall outside of the V-mask arms. These arms take the place of the upper and lower control limits.



The chart in Figure III.5 indicates that a process shift occurred around the time of sample 14 or 15. Due to the nature of this chart, the shift was not detected until sample 23 was plotted. When the V-mask was positioned on prior data points, all samples fell within the control limits, so there was no indication of an out-of-control situation.

In comparison, an Individual and Moving Range (X, MR) plot of the same data (Figure III.6) does not detect the process shift until sample 27.

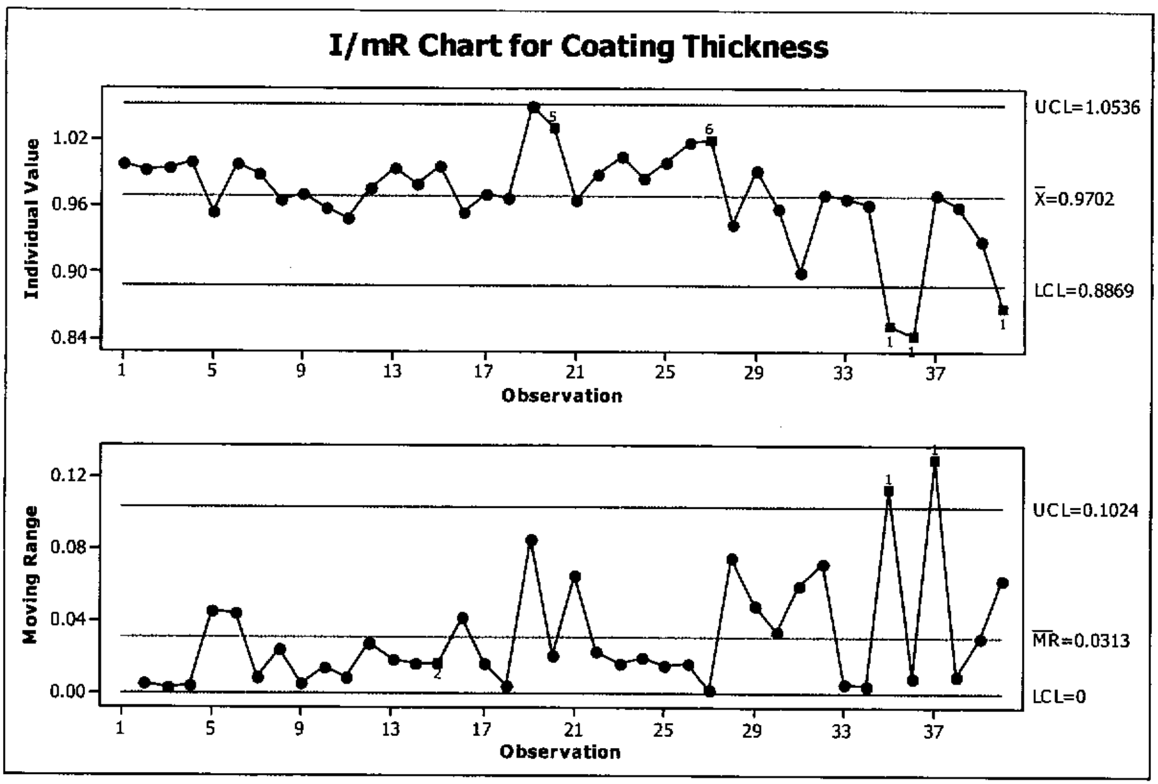


Figure III.6: *X, MR* Chart

A tabular CUSUM is an alternative to the V-mask approach. See Montgomery (1997) for a discussion of this procedure.

EWMA (Exponentially Weighted Moving Average) Charts

An EWMA Chart plots moving averages of past and current data in which the values being averaged are assigned weights that decrease exponentially from the present into the past.[30] Consequently, the average values are influenced more by recent process performance.[31] The exponentially weighted moving average is defined by the equation:

$$z_t = \lambda x_t + (1-\lambda) z_{t-1}$$

where λ is the weighting constant $0 < \lambda < 1$,

t is an index number ($t = 1....$),

x_t is the current sample value, and

z_t is the current weighted moving average.

An initial value, z_0 must be estimated to start the process with the first sample.

Through recursive substitution, successive values of Z_t can be determined from the equation:

$$z_t = \lambda \sum_{i=0}^{t-1} (1-\lambda)^i x_{t-i} + (1-\lambda)^t z_0 \qquad \text{for } 0 < \lambda < 1$$

The value of λ is determined from tables or graphs based on Average Run Length (ARL) performance. Some authors also consider control limit widths other than three-sigma when designing an EWMA chart. But, current literature indicates that this approach may not necessary. The EWMA chart becomes an $\overline{X}$ chart for λ = 1.0. See Montgomery (1997) and Wheeler (1995) for detailed discussions.

The advantage of this chart is its ability to efficiently detect small process mean shifts, typically less then 1.5 sigma, and it can be used with an autocorrelated process[32] with a slowly drifting mean.

Its disadvantage is its inability to efficiently detect large changes in the process mean. In situations where large process mean shifts are expected, the Shewhart control chart is recommended.

A common use of the EWMA is in the chemical industry where large day-to-day fluctuations are common but may not be indicative of the lack of process predictability.

Figures III.7 and III.8 are EWMA and *X, MR* plots of the same data. The EWMA chart detects a mean shift at sample 29, but there is no indication of this shift on the *X, MR* chart

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30 In contrast, the CUSUM chart gives equal weight to the previous data.

31 Another type of time weighted control chart is the Moving Average chart (MA chart). This approach is based on a simple, unweighted moving average. See Montgomery (1997).

32 See Appendix A.

Valid signals occur only in the form of points beyond the control limits.[33] Other rules used to evaluate the data for non-random patterns (see Chapter II, Section B) are not reliable indicators of out-of-control conditions.

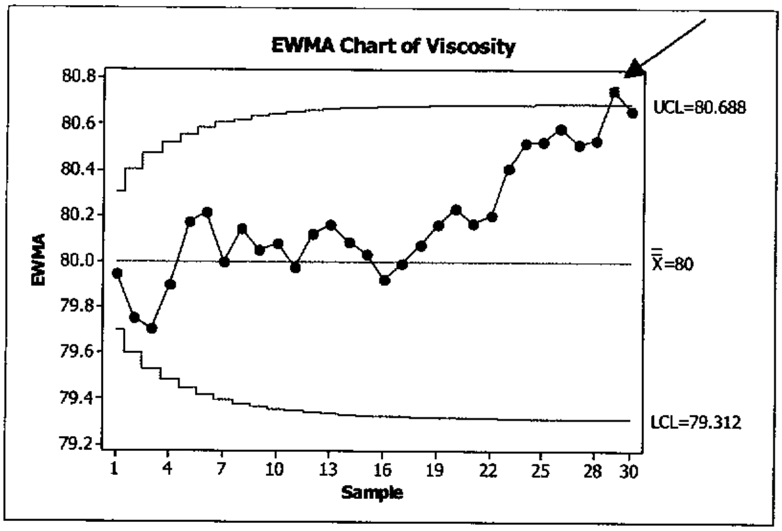


Figure III.7: EWMA Chart of Viscosity

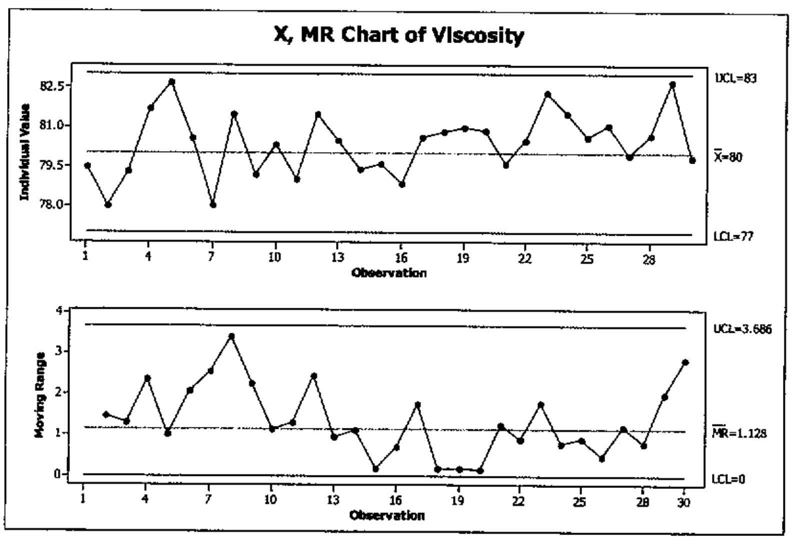


Figure III.8: *X, MR* Chart of Viscosity

EWMA and CUSUM essentially are equivalent in their ability to detect the presence of assignable causes that result in a small shift in the mean. However, the EWMA also can be used to forecast a "new" process mean for the next time period. These charts can be useful to signal a need to

33 Because moving averages are involved, the points being plotted are correlated (dependent) and therefore detection of special causes using pattern analysis is not appropriate since they assume independence among the points.

adjust (maintain) a process. But they are not appropriate as tools for process improvement (see Wheeler (1995)).

Multivariate forms of these charts, MCUSUM and MEWMA, have been developed. See Lowery et al. (1992) and Lowry and Montgomery (1995).

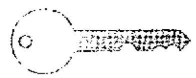
Non-Normal Charts

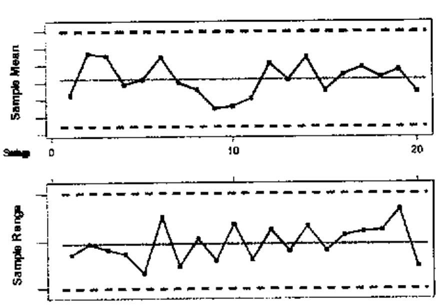
If the underlying distribution of a process is known to be non-normal, there are several approaches that can be used:

- Use the standard Shewhart control charts with appropriate sample size.
- Use adjustment factors to modify the control limits to reflect the non-normal form.
- Use a transformation to convert the data into a (near) normal form and use the standard charts.
- Use control limits based on the native non-normal form.

The approach which is used depends on the amount the process distribution deviates from normality and specific conditions related to the process.

Shewhart Control Charts





Although the sensitivity and risks associated with the standard control charts have been analyzed by assuming the process distribution was normal, Shewhart's development was not based on an assumption of normality. His goal was to develop a tool useful for the economic control of quality. Shewhart control charts can be used for all processes. However, as the process distribution deviates from normality, the sensitivity to change decreases, and the risk associated with the Type I error increases.

For many non-normal process distributions, the Central Limit Theorem can be used to mitigate the effect of non-normality. That is, if a sufficiently large subgroup size is used,[34] the Shewhart control chart can be used with near normal sensitivity and degree of risk.

[34] For example, see Wheeler (1995).

Central Limit Theorem

Let $X_1,\ldots,X_n$ be a set of n independent random variates from the same *arbitrary* probability distribution $P(x_1,\ldots,x_n)$ with mean μ_X and variance σ_X.

Consider the average $\bar{X}_n = \frac{\sum_{i=1}^{n} X_i}{n}$.

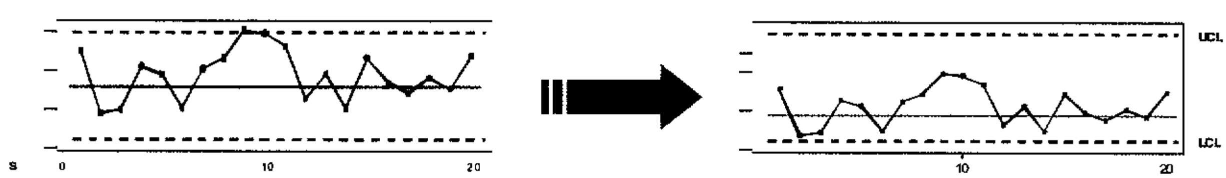
The distribution of $\bar{X}_n$ approaches the normal distribution $N\left(\mu_X, \frac{\sigma_X}{n}\right)$ as $n \to \infty$

The "rule of thumb" is that the range chart should be used with subgroups of size fifteen or less. The standard deviation chart can be used for all subgroup sizes.

Adjustment Factors

When a large subgroup size is not possible, the control limits of the Shewhart control charts can be modified using adjustment factors to compensate for the effect of the non-normality. Since non-normal distributions are either asymmetric, have heavier tails than the normal distribution, or both, use of the standard ± 3 sigma control limits can increase the risk of false alarms, especially if pattern analysis for special causes is used.

In this approach the non-normal distributional form is characterized by its skewness or kurtosis or both. Tabled or algorithmic correction factors are then applied to the normal control limits.[35]



This approach requires an initial capability study with a sample size sufficiently large to effectively capture the non-normal form.

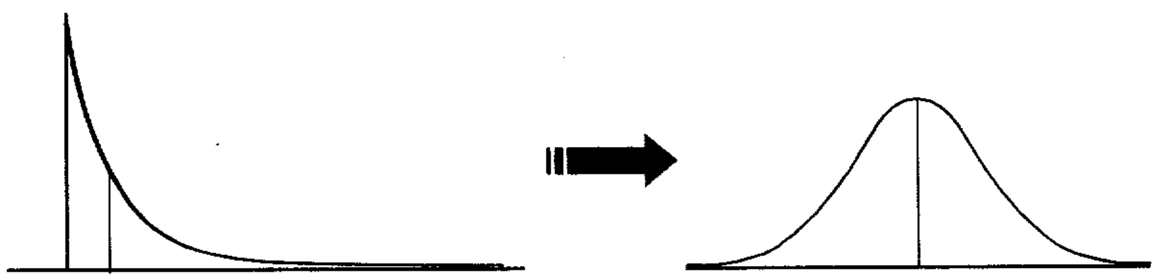
[35] For example see: Burr, I. W., (1967), Chan, L.K., and Cui, Heng, J, and (2003) Pham, H., (2001).

For this and the following approaches, the process should be studied periodically to verify that the distributional form has not changed. Any significant change in the distribution is an indicator that the process is being affected by special causes.

Transformations

An alternative to the adjustment factors is to convert the data instead of the control limits. In this approach, a transformation is determined which transforms the non-normal process distribution into a (near) normal distribution. Examples of transformations[36] used in these situations are the Johnson family of transformations and the Box-Cox transformations.

The selected transformation is then used to transform each datum point and the standard Shewhart control chart methodologies are used on the converted data.

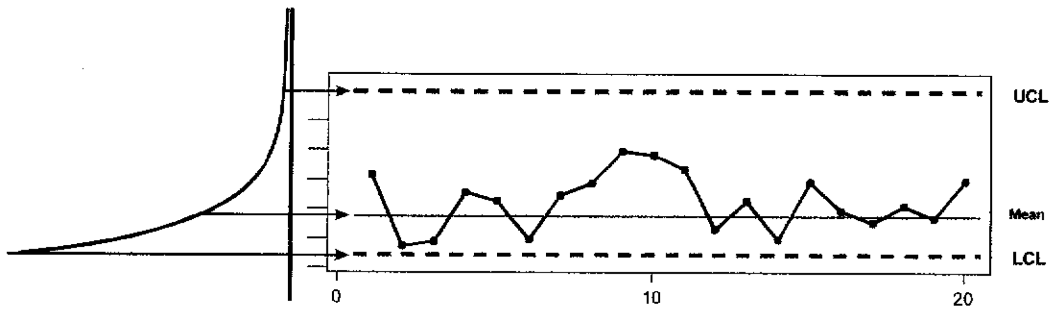


For this approach to be effective, the transformation must be valid. This typically requires a capability study with a sample size sufficiently large to effectively capture the non-normal form. Also, because the transformations tend to be mathematically complex, this approach is only effective and efficient when implemented using a computer program.

Non-Normal Form

There are situations when the above approaches are not easily handled. Examples of these situations occur when the process distribution is highly non-normal and the sample size cannot be large, e.g., when tracking equipment reliability. In these situations a control chart can be developed using the non-normal form directly to calculate the chart control limits.

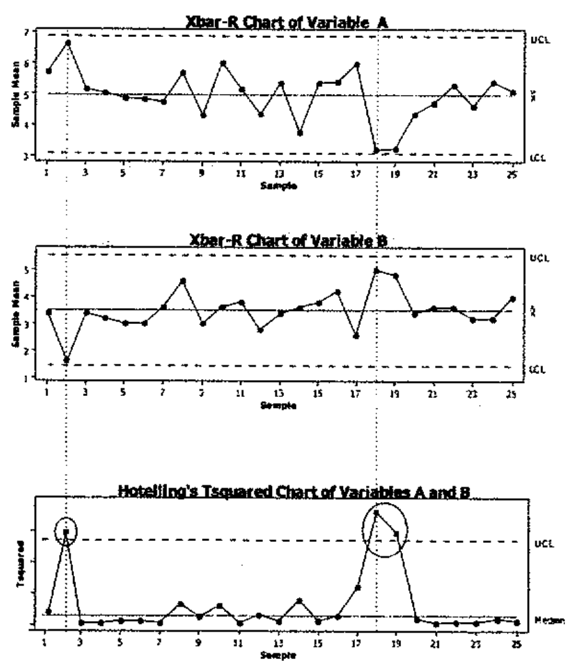
[36] For example, see Johnson (1949) and Box and Cox (1964).



In the example of tracking equipment reliability, a Time-to-Failure Chart with a subgroup size of one can be used. The control limits are based on the exponential distribution with parameter θ equal to the mean time between failures (MTBF). In general, control limits for this approach are selected to be the 0.135 and 99.865 percentile points of the underlying distribution.

Like the other approaches above, for this approach to be effective, it typically requires a capability study with a sample size sufficiently large to capture the non-normal form. Advantages of this approach are that the data can be plotted without complex calculations and it provides more exact control limits than adjustment factors.

Multivariate



Multivariate charts are appropriate when it is desired to simultaneously control two or more related characteristics that influence the performance of a process or product. Their advantage is that the combined effect of all variables can be monitored using a single statistic. For instance, the combined effects of pH and temperature of a part washing fluid may be linked to part cleanliness measured by particle count. A multivariate chart provides a means to detect shifts in the mean and changes in the parameter relationships.

A correlation matrix of variables can be used to test whether a multivariate control chart could be useful. For the multivariate approach to be viable the matrix entries should indicate that the variables are sufficiently correlated.

Three of the most popular multivariate control chart statistics are Hotelling's T^2, the Multivariate Exponentially-Weighted Moving Average (MEWMA) and the Multivariate Cumulative Sum (MCUSUM).

A multivariate chart reduces Type I error, i.e., false out-of-control signals are less likely to occur compared to using univariate charts to make decisions separately for each variable.

The simplicity of this approach is also its disadvantage. An out-of-control condition can be detected using a single statistic but the analysis of the charted results may not tell which variable caused it. Additional analysis using other statistical tools may be required to isolate the special cause(s). See Kourti and MacGregor (1996).

Multivariate charts are mathematically complex, and computerized implementation of these methods is essential for practical application. It is important, however, that the use of appropriate techniques for estimating dispersion statistics be verified. See Wheeler (1995), Montgomery (1997) and current literature such as Mason and Young (2001), for detailed discussions of multivariate control charts.

Other Charts

In Chapter I, Section E, a Case 3 process was defined as one not in statistical control but acceptable to tolerance. Special causes of variation are present, the source of variation is known and predictable but may not be eliminated for economic reasons. However, this predictability of the special cause may require monitoring and control. One method to determine deviations in the predictability of special cause variation is the Regression chart.

Regression Control Charts

Regression charts are used to monitor the relationship between two correlated variables in order to determine if and when deviation from the known predictable relationship occurs. These charts originally were applied to administrative processes but they have also been used to analyze the correlation between many types of variables.

Regression charts track the linear correlation between two variables, for example:

- Product cost versus weight.
- Throughput versus machine cycle time (line speed).
- Temperature versus pressure.
- Dimensional change relative to tooling cycles.

For example, if a tool has constant wear relative to each cycle of the process, a dimensional feature such as diameter (Y) could be predicted based on the cycles (X) performed. Using data collected over time this linear relationship can be modeled as

$$Y = b_0 + b_1 X$$

When X equals zero cycles, the predicted Y is equal to b_0. So b_0 is the predicted dimension from a tool never used.

b_0 and b_1 are estimated using the equations for simple linear regression. The chart is constructed by drawing the line $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$ which is the estimate for $Y = b_0 + b_1 X$ and computing the 95% or 99% predictive interval. The predictive limits computed are curved lines with the tightest point at $\overline{X}$. Often they are replaced with the $\hat{Y} \pm 3s$ in order to tighten the control limits at each extreme for X.

Points that exceed the control limits indicate tooling which has a tool life which is significantly different from the base tool life. This can be advantageous or detrimental depending on the specific situation.

A line is only one type of correlation between variables. Regression charts can be applied to any relationship for which the mathematical model can be determined.

Care should be taken in making predictions (extrapolating) outside of the range of the original observations. The accuracy of the regression model for use outside of this range should be viewed as highly suspect. Both the prediction interval for future values and the confidence interval for the regression equation become increasingly wide. Additional data may be needed for model validation.

Discussion on confidence intervals can be found in Hines and Montgomery (1980).

Residual Charts

An alternative approach to the Regression Chart is to chart the residual values. From the regression equation, the residual value (ε) is $Y - \hat{Y}$. A chart of the residual values could be treated in the same manner as an Individuals chart with $\overline{X}$ equal to zero.

The Residuals Chart and the Regression Chart are technically equivalent and differ only in their presentation.

This approach would be more useful and intuitive when the variable relationships are more complex.

Autoregressive Charts

Control chart methods generally assume that the data output from a process are independent and identically distributed. For many processes this assumption is not correct. Data from a *time series,* data taken sequentially in time, are often serially dependent. These types of processes have output that are *autocorrelated* and analysis with standard charting methods may result in erroneous conclusions.

One common approach to contend with serial dependency is to take samples far enough apart in time that the dependency is not apparent. This often works in practice but its effectiveness relies on postponing sample collection and may extend the sampling interval longer than is appropriate. Also, this approach ignores information which may be useful or even necessary for accurate prediction in order to utilize techniques which were not designed for that type of data.

Autoregressive Moving Average (ARMA) is a method of analysis for data from a time series and it results in the prediction of future observations based on their dependency on past observations. Processes which drift, walk or cycle through time are good candidates for time series analysis and an ARMA method may be appropriate.

The autoregressive (AR) model is defined by

$$X_i = \xi + \phi_1 X_{i-1} + \phi_2 X_{i-2} \ldots + \varepsilon_i \quad \text{(AR)}$$

The current value observed is equal to a constant, a weighted combination of prior observations and a random component.

The moving average (MA) model is defined by

$$X_i = \mu - \theta_1 \varepsilon_{i-1} - \theta_2 \varepsilon_{i-2} \ldots + \varepsilon_i \quad \text{(MA)}$$

The current value observed is equal to a constant, plus a weighted combination of prior random adjustments and a random component.

The ARMA model is a combination of the AR and MA models.

$$X_i = \xi + \phi_1 X_{i-1} - \theta_1 \varepsilon_{i-1} + \phi_2 X_{i-2} - \theta_2 \varepsilon_{i-2} \ldots + \varepsilon_i \quad \text{(ARMA)}$$

Box and Jenkins defined the terminology ARMA(p,d,q) to define subsets of the full ARMA model, where p is the number of autoregressive parameters, d the number of times the data is differenced (defined below), and q is the number of moving average parameters.

So, ARMA(1,0,0) is the first order AR model without differencing

$$X_i = \xi + \phi_1 X_{i-1} + \varepsilon_i .$$

ARMA(0,0,1) is the first order MA model without differencing

$$X_i = \mu - \theta_1 \varepsilon_{i-1} + \varepsilon_i .$$

And ARMA(1,1,1) is the first order ARMA model differenced once

$$Y_i = \xi + \phi_1 Y_{i-1} - \theta_1 \varepsilon_{i-1} + \varepsilon_i .$$

For the first order AR and ARMA models the parameter ϕ must be in the interval $-1 < \phi < 1$ for the model to be *stationary*, i.e., does not diverge to infinity. (There are similar restrictions for the ϕ's in the higher order models.) For processes that are not stationary, the data will need to be *differenced*. Differencing removes the serial dependence between an observation and another lagged observation.

$$Y_i = X_i - X_{i-k}$$

The differenced observation is equal to the current observation minus the observation made k samples prior. The data should only be differenced if the model is not stationary. Most data from manufacturing processes will not need differencing. The processes do not diverge to infinity.

The next step is to determine the number of autoregressive and moving average parameters to include in the model. Typically the number of ϕ's or θ's needed will not be more than two. ARMA(1,d,0), ARMA(2,d,0), ARMA(1,d,1), ARMA(2,d,1), ARMA(1,d,2), ARMA(2,d,2), ARMA(0,d,1), ARMA(0,d,2) are the common combinations and it is feasible to estimate them all before selecting the best.

To estimate the parameters use Non-Linear Estimation.

Once the model is determined and stationary, and the parameters are estimated then the next observation can be predicted from past observations. For example (ARMA(1,0,1)):

$$\hat{X}_t = \xi + \phi_1 X_{t-1} - \theta_1 \varepsilon_{t-1}$$

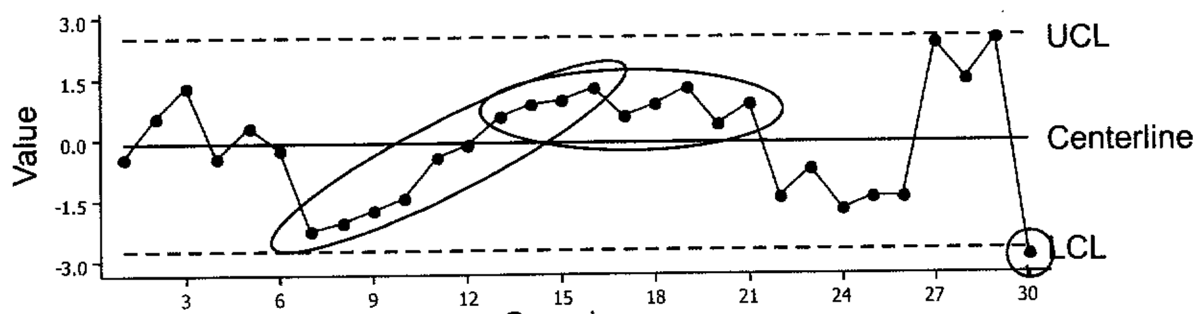
The residual is calculated by

$$\varepsilon_t = X_t - \hat{X}_t$$

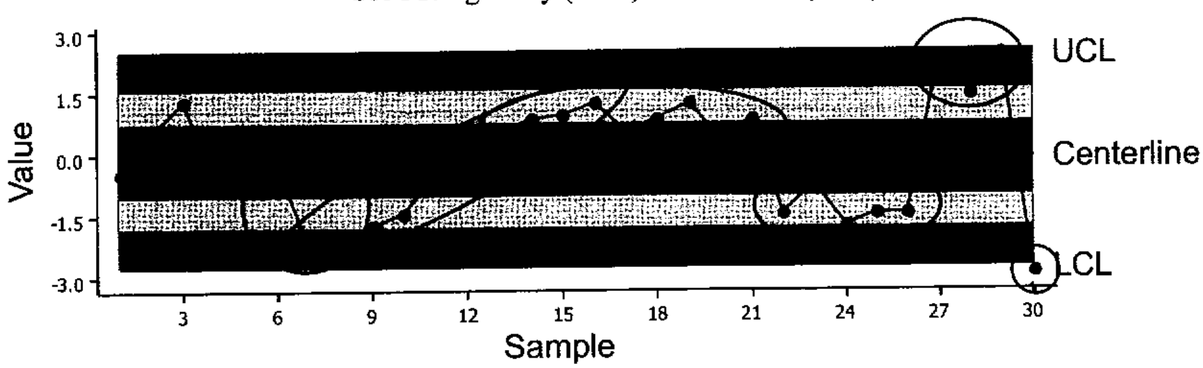
and the values for ε will be independent normally distributed random variables and may be analyzed using an Individuals Chart or Residuals Chart. For a more complete discussion see Box, Jenkins and Reinsel (1994).

Zone Charts

Chapter II, Section B, Table II.1 provides various rules for detecting out-of-control signals. The first four rules can be easily implemented with manual control charts, but the latter rules do not lend themselves to rapid visual identification since they require the determination of the number of standard deviations a plotted point is from the centerline. This can be aided by dividing the control chart into "zones" at 1, 2, and 3 standard deviations from the centerline.



These zones are sometimes referred to as "sigma" zones (sigma here is the standard deviation of the distribution of the sample averages, not the individual values). The zones assist in the visual determination of whether a special cause exists using one or more of the tabled criteria. See Montgomery (1997) and Wheeler (1995).

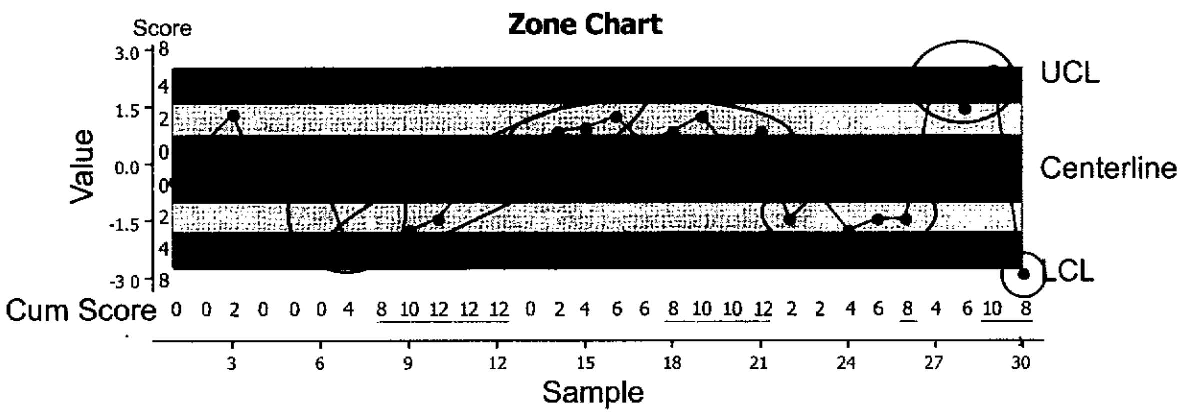


This division of the control chart can be coupled with *run sums analysis* of control chart to produce the Zone Control Chart. The run sums control chart analysis was introduced by Roberts (1966) and studied further by Reynolds (1971). This approach assigns a score to each zone. The score α_i, assigned to the region R_{+i} is nonnegative; and the score, β_i assigned to the region R_{-i} is nonpositive. A typical set of scores are:

Zone	Score
$\left[\mu_X, \mu_X + \sigma_{\bar{X}}\right)$	0 or +1
$\left[\mu_X + \sigma_{\bar{X}}, \mu_X + 2\sigma_{\bar{X}}\right)$	+2
$\left[\mu_X + 2\sigma_{\bar{X}}, \mu_X + 3\sigma_{\bar{X}}\right)$	+4
$\left[\mu_X + 3\sigma_{\bar{X}}, \infty\right)$	+8

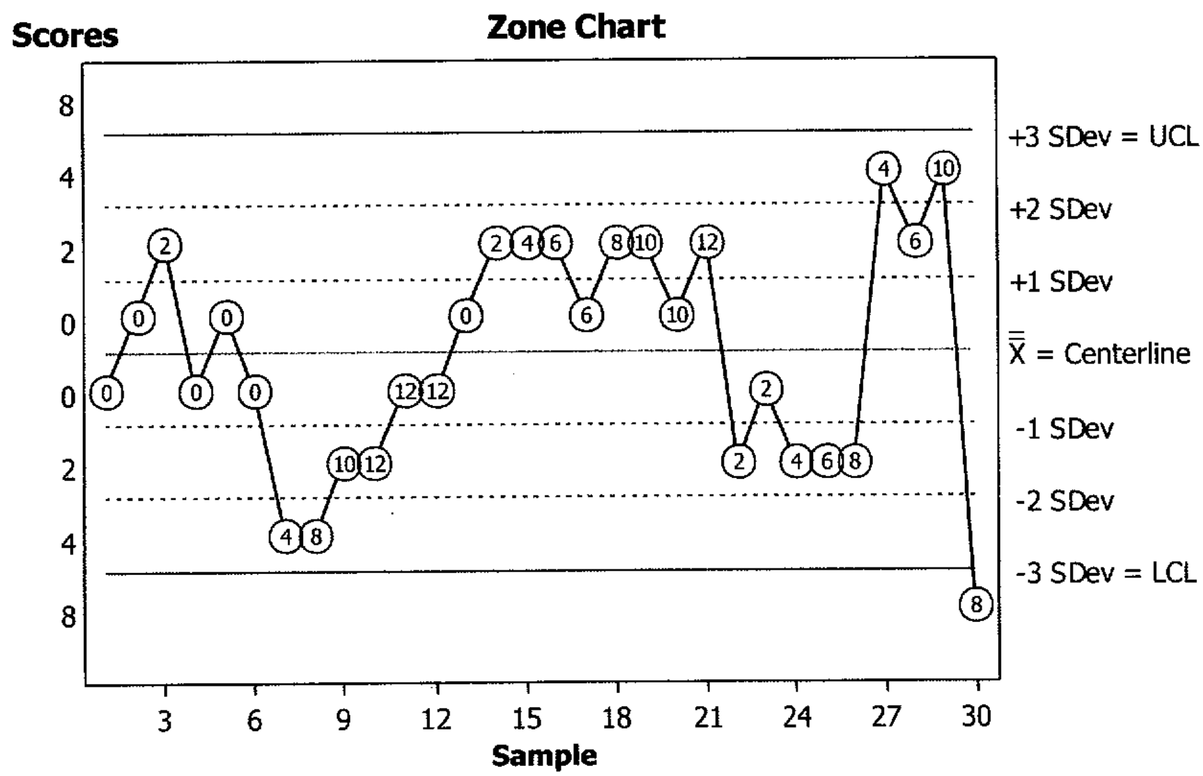
The four regions placed symmetrically below the centerline are assigned the corresponding negative scores.

A zone control chart is a hybrid between an $\bar{X}$ (or Individuals) chart and a CUSUM chart. It analyzes a cumulative score, based on the zones. The cumulative score is the absolute value of the sum of the scores of the zones in which the points are plotted. Every time the centerline is crossed the cumulative score is reset to zero.



A point is out of control if its cumulative score is greater than or equal to 8. Thus, the analyst does not need to recognize the patterns associated with non-random behavior as on a Shewhart chart. With the scoring of 0, 2, 4, 8 this method is equivalent to the standard criteria 1, 5, and 6 for special causes in an $\bar{X}$ (or Individuals) chart and is more stringent than criterion 8. With the scoring of 1, 2, 4, 8 this method is equivalent to the standard criteria 1, 2, 5, and 6 for special causes in an $\bar{X}$ (or Individuals) chart and is more stringent than criteria 7 and 8. As shown in the figure above, trends (criterion 3) can also be detected depending on the start and stop of the trend.

Zone control charts can be modified to eliminate the point-plotting process; the points are plotted in the zone not to a scale. Thus, one standard zone control chart can fit most needs; when to act on a process is determined by the charting procedure.



The zone chart can be used with a weighting scheme to provide the sensitivity needed for a specific process. For example, one set of weights (scores) can be used during the initial phase for detecting special causes. Then the weights could be changed when the process is in control and it is more important to detect drift.

The efficiency of the zone control chart is demonstrated by comparing its average run lengths with those of standard control tests. For the chart divided into scores of 0, 2, 4, and 8, the zone control chart performs as well as or better than Shewhart charts (see Davis et al. (1990)).

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CHAPTER IV

Understanding Process Capability and Process Performance for Variables Data

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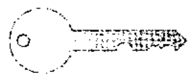
Introduction

The output of a stable process can be described by its statistical distribution. The process must be stable (in statistical control) in order for the distribution to be useful for predicting future results.[37] A distribution is described in terms of characteristics (statistics) that are calculated from measurements of samples taken from the process.

The statistics of most frequent interest are estimates of distribution location (or center) and spread relative to the customer requirements. Typically, the location is estimated by the sample mean or sample median. Spread usually is estimated using the sample range or sample standard deviation.

Process centering and spread interact with respect to producing an acceptable product. As the distribution moves off center, the "elbow room" available to accommodate process variation (spread) is reduced. A shift in process location, an increase in process spread or a combination of these factors may produce parts outside the specification limits. A process with such a distribution would not be qualified to meet the customer's needs.



This section addresses some of the techniques for evaluating process capability and performance with respect to product specifications. In general, it is necessary that the process being evaluated be stable (in statistical control). A discussion of process variation and the associated capability indices has little value for unstable processes. However, reasonable approaches have been developed to assess the capability of processes exhibiting systematic special causes of process variation, such as tool wear (see Spiring, F. A. (1991)).

In addition, it is generally assumed that the individual readings from the subject processes have a distribution that is approximately normal.[38] This section will discuss only the more popular indices and ratios:

- Indices of process variation-only, relative to specifications: C_p, and P_p.
- Indices of process variation and centering combined, relative to specifications: C_{pk}, and P_{pk}.
- Ratios of process variation-only, relative to specifications: *CR* and *PR*.

NOTE: Although other indices are not discussed in this manual, see Appendix D and References for information on other indices.

Finally, this section describes the conditions and assumptions associated with these process measures and concludes with a suggestion as to how these measures might be applied toward enhancing process understanding within the framework of continual process improvement.

[37] See Chapter I, Sections C-F.

[38] For non-normal distributions and autocorrelated processes see Chapter IV, Section B.

This manual recognizes both the misunderstanding and controversy surrounding the fundamental concepts and definitions regarding process "Control", "Capability", and "Performance". It is not the purpose of this manual to fully resolve these issues, but to expose and discuss them to an extent that allows each reader the opportunity to develop a better understanding of them in order to provide value and knowledge for continual process improvement.

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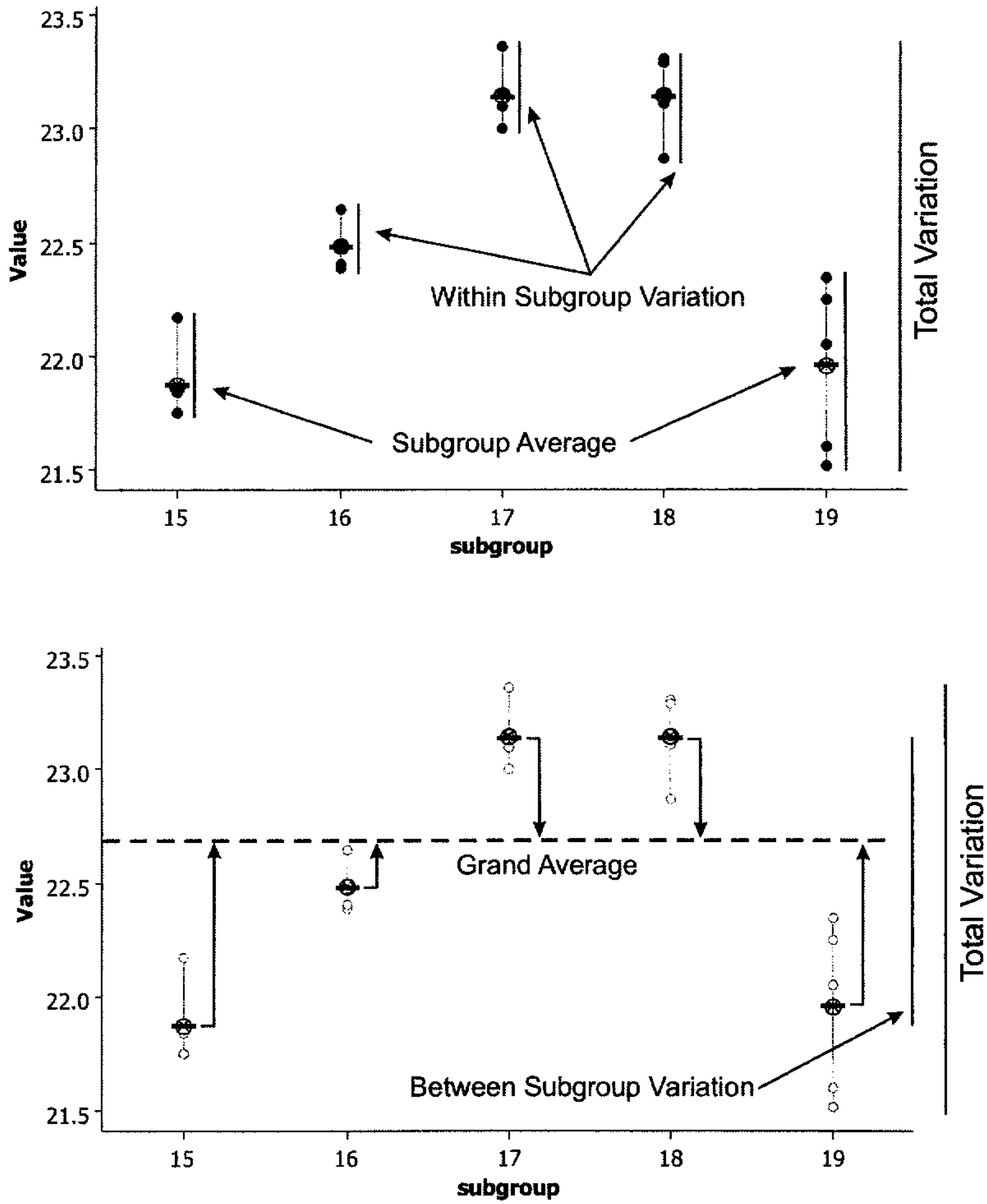


Figure IV.1: Within- and Between-Subgroup Variation

CHAPTER IV - Section A

Definitions of Process Terms

Process variation has various aspects:

- *Inherent Process Variation* – That portion of process variation due to common (systematic) causes only.
- *Within-subgroup Variation* (σ_C) – This is the variation due only to the variation within the subgroups. If the process is in statistical control this variation is a good estimate of the inherent process variation. It can be estimated from control charts by $\bar{R}/d_2$ or $\bar{s}/c_4$.
- *Between-subgroup Variation* – This is the variation due to the variation between subgroups. If the process is in statistical control this variation should be zero.
- *Total Process Variation* (σ_P) – This is the variation due to both within-subgroup and between-subgroup variation. If the process is not in statistical control the total process variation will include the effect of the special cause(s) as well as the common causes. This variation may be estimated by s, the sample standard deviation, using all of the individual readings obtained from either a detailed control chart or a process study: $\sigma_P = s = \sqrt{\sum_i^n \frac{(x_i - \bar{x})^2}{n-1}}$ where x_i is an individual reading, $\bar{x}$ is the average of the individual readings, and n is the total number of individual readings.
- *Process Capability* — The $6\hat{\sigma}$ range of inherent process variation, for statistically stable processes only, where $\hat{\sigma}$ is usually estimated by $\bar{R}/d_2$ or $\bar{s}/c_4$.
- *Process Performance* — The $6\hat{\sigma}$ range of total process variation, where $\hat{\sigma}$ is usually estimated by s, the total process standard deviation.

If the process is in statistical control the process capability will be very close to the process performance. A large difference between the capability and performance $\hat{\sigma}$ indicates the presence of a special cause(s).

Process Measures for Predictable Processes

Indices – Bilateral Tolerances

This section discusses commonly used indices where the specification has both an upper and lower limit.[39]

CAUTION: The indices discussed below are *valid* only when the process is stable (*in statistical control*). If the process is not in statistical control then these indices can be very misleading, as can be seen by Figure IV.4.

C_p

C_p: This is a capability index. It compares the process capability to the maximum allowable variation as indicated by the tolerance. This index provides a measure of how well the process will satisfy the variability requirements. C_p is calculated by $C_p = \dfrac{USL - LSL}{6\sigma_C} = \dfrac{USL - LSL}{6\left(\bar{R}/d_2\right)}$

C_p is not impacted by the process location. This index can be calculated only for two-sided (bilateral) tolerances.

C_{pk}

C_{pk}: This is a capability index. It takes the process location as well as the capability into account. For bilateral tolerances C_{pk} will always be less than or equal to C_p.

$$C_{pk} \le C_p$$

C_{pk} will be equal to C_p only if the process is centered.

C_{pk} is calculated as the as the minimum of *CPU* or *CPL* where:

$$CPU = \frac{USL - \bar{\bar{X}}}{3\sigma_C} = \frac{USL - \bar{\bar{X}}}{3\left(\bar{R}/d_2\right)} \quad \text{and}$$

$$CPL = \frac{\bar{\bar{X}} - LSL}{3\sigma_C} = \frac{\bar{\bar{X}} - LSL}{3\left(\bar{R}/d_2\right)}$$

C_{pk} and C_p should always be evaluated and analyzed together. A C_p value significantly greater than the corresponding C_{pk} indicates an opportunity for improvement by centering the process.

39 As discussed in Chapter II, Section A, process analysis requires that the data have been collected using measurement system(s) that are consistent with the process and have acceptable measurement system characteristics.

P_p

P_p: This is a performance index. It compares the process performance to the maximum allowable variation as indicated by the tolerance. This index provides a measure of how well the process will satisfy the variability requirements. P_p is calculated by

$$P_p = \frac{USL - LSL}{6\sigma_P} = \frac{USL - LSL}{6s}$$

P_p is not impacted by the process location.

P_{pk}

P_{pk}: This is a performance index. It takes the process location as well as the performance into account. For bilateral tolerances P_{pk} will always be less than or equal to P_p. P_{pk} will be equal to P_p only if the process is centered.

$$P_{pk} \leq P_p$$

P_{pk} is calculated as the as the minimum of PPU or PPL where:

$$PPU = \frac{USL - \bar{\bar{X}}}{3\sigma_P} = \frac{USL - \bar{\bar{X}}}{3s} \quad \text{and}$$

$$PPL = \frac{\bar{\bar{X}} - LSL}{3\sigma_P} = \frac{\bar{\bar{X}} - LSL}{3s}$$

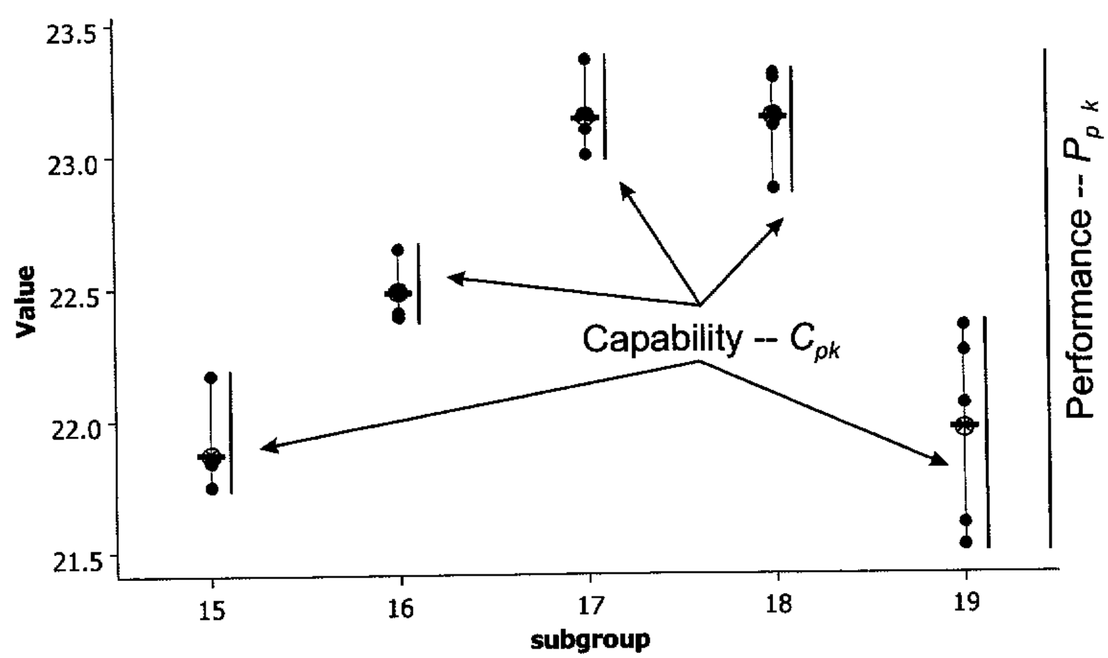


Figure IV.2: C_{pk} and P_{pk} Comparison

P_{pk} and P_p should always be evaluated and analyzed together. A P_p value significantly greater than the corresponding P_{pk} indicates an opportunity for improvement by centering the process.

If the process is in statistical control the process capability will be very close to the process performance. A large difference between the C and P indices indicate the presence of a special cause(s). See Figure IV.3 and IV.4.

CR

CR: This is the capability ratio and is simply the reciprocal of C_p;

$$CR = \frac{1}{C_p}.$$

PR

PR: This is the performance ratio and is simply the reciprocal of P_p;

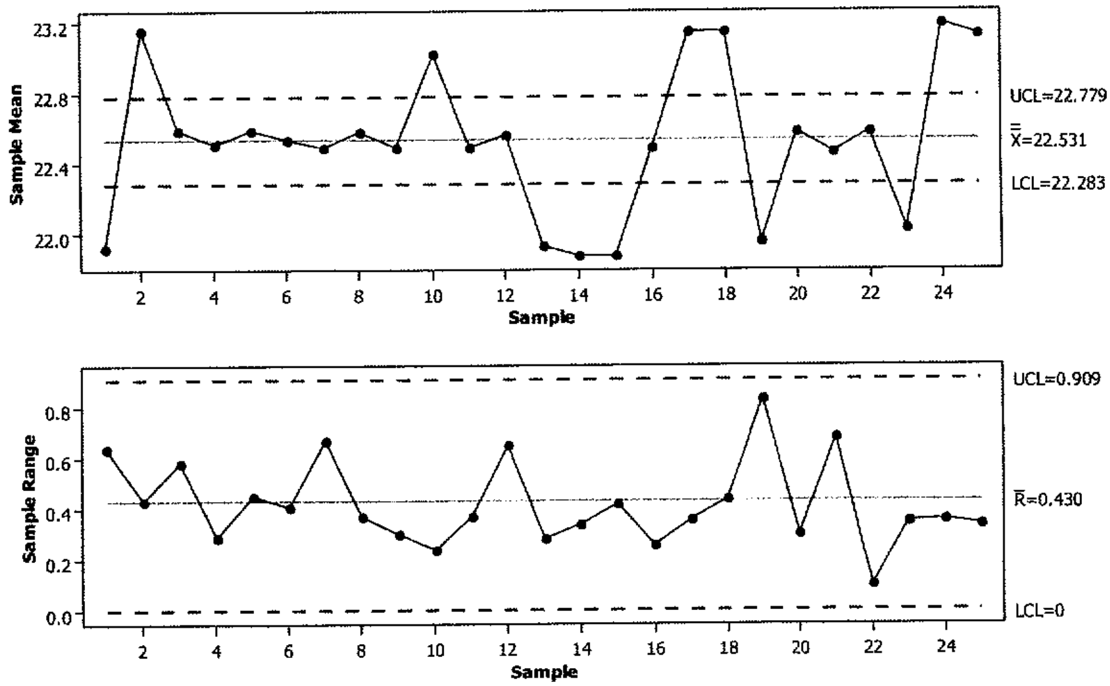
$$PR = \frac{1}{P_p}.$$

NOTE: Example calculations for all of these measures are shown in Appendix F.

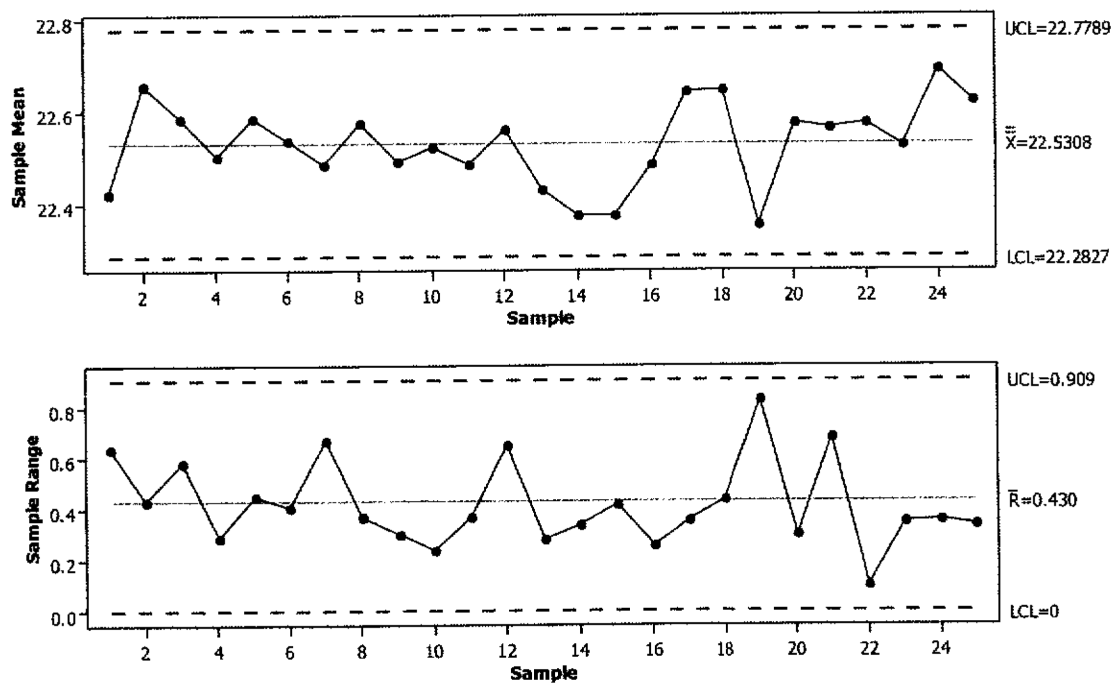
PPM

A parts-per-million (ppm) nonconformance rate is sometimes used as a supplemental measure of process capability. To estimate the nonconformance rate using capability index information, a probability distribution of the data must be defined. While the normal distribution often is used for this purpose, this is an assumption that should be validated using a goodness-of-fit test before proceeding further. The nonlinear relationship between the capability index and the proportion nonconforming should be understood in order to make correct inferences (see Wheeler (1999) for a detailed discussion of this subject).

Xbar - R Chart: Immature Process



Xbar - R Chart: Predictable Process

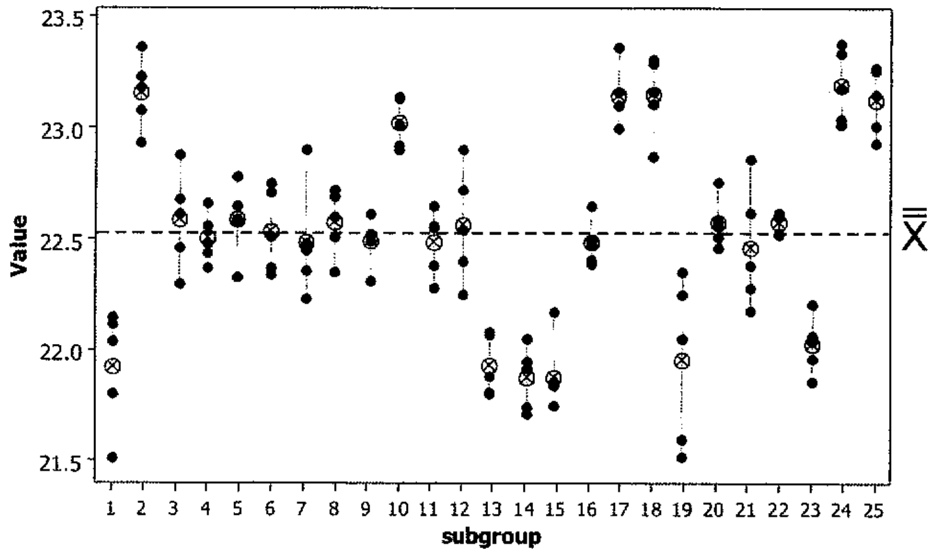


Note that the Range charts are identical since the within subgroup variation is the same for both processes

Figure IV.3: Comparison between a Predictable and Immature Process

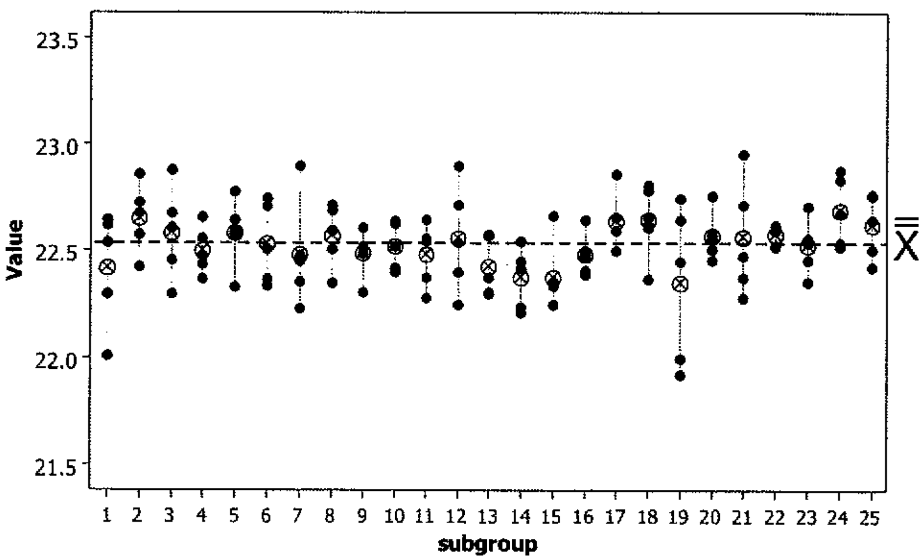
Immature Process -- out of statistical control

Ppk = *0.71*
Cpk = *1.80*



Process below has same within subgroup variation
as above but no between subgroup variation

Ppk = *1.71*
Cpk = *1.80*



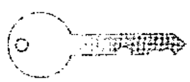
Process is in statistical control

Figure IV.4: *Cpk* and *Ppk* Values Produced by a Predictable and Immature Process

Indices – Unilateral Tolerances

This section discusses commonly used indices where the specification has either an upper or lower limit but not both.

C_p



C_p: This is a capability index. It compares the process capability to the maximum allowable variation as indicated by the tolerance. This index has no meaning for unilateral tolerances.

If the product characteristic has a physical limit (e.g., flatness cannot be less than zero), a C_p could be calculated using the physical limit (0.0) as a surrogate lower limit. But this number will not have the same relationship to C_{pk} as it does in the bilateral case.

C_{pk}

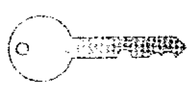
C_{pk}: This is a capability index. It takes the process location as well as the capability into account. With unilateral tolerances with a physical limit, C_{pk} can be less than, equal to or greater than C_p.

C_{pk} is directly related to the proportion nonconforming produced by the process. It is equal to CPU or CPL depending whether the tolerance is an USL or a LSL where:

$$CPU = \frac{USL - \bar{\bar{X}}}{3\left(\bar{R}/d_2\right)}$$

$$CPL = \frac{\bar{\bar{X}} - LSL}{3\left(\bar{R}/d_2\right)}$$

P_p



P_p: This is a performance index. It compares the process performance to the maximum allowable variation as indicated by the tolerance. This index has no meaning for unilateral tolerances.

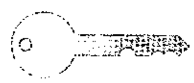
If the product characteristic has a physical limit (e.g., flatness cannot be less than zero), a P_p could be calculated using the physical limit (0.0) as a surrogate lower limit. But this number will not have the same relationship to P_{pk} as it does in the bilateral case.

P_{pk}

P_{pk} is directly related to the proportion nonconforming produced by the process. It is equal to *PPU* or *PPL* depending whether the tolerance is an *USL* or a *LSL* where:

$$PPU = \frac{USL - \overline{\overline{X}}}{3s}$$

$$PPL = \frac{\overline{\overline{X}} - LSL}{3s}$$



An alternate notation for P_{pk} in the case of unilateral tolerances is P_{pku} or P_{pkl} depending on whether the limit is an *USL* or *LSL*.

CR

CR: This is the capability ratio and is simply the reciprocal of C_p. As such, this index has no meaning for unilateral tolerances.

PR

PR: This is the performance ratio and is simply the reciprocal of P_p. As such, this index has no meaning for unilateral tolerances.

NOTE: Example calculations for all of these measures are shown in Appendix F.

CHAPTER IV - Section B

Description of Conditions

It is appropriate to point out that process variation and process centering are two separate process characteristics. Each needs to be understood separately from the other. To assist in this analysis it has become convenient to combine the two characteristics into indices, such as C_p, C_{pk} or P_p, P_{pk}. These indices can be useful for:

- Measuring continual improvement using trends over time.
- Prioritizing the order in which processes will be improved.

The capability index, C_{pk}, is additionally useful for determining whether or not a process is capable of meeting customer requirements. This was the original intent of the capability index. The performance index, P_{pk}, shows whether the process performance is actually meeting the customer requirements. For these indices (as well as all of the other process measures described in Chapter IV, Section A) to be effectively used, the *CONDITIONS* which surround them must be understood. If these conditions are not met, the measures will have little or no meaning and can be misleading in understanding the processes from which they were generated. The following three conditions are the minimum that must be satisfied for all of the capability measures described in Section A:

- The process from which the data come is statistically stable, that is, the normally accepted SPC rules must not be violated.
- The individual measurements from the process data form an approximately normal distribution.[40]
- The specifications are based on customer requirements.

Commonly, the computed index (or ratio) value is accepted as the "true" index (or ratio) value; i.e., the influence of sampling variation on the computed number is discounted. For example, computed indices C_{pk} of 1.30 and 1.39 can be from the same stable process simply due to sampling variation.

See Bissell, B.A.F. (1990), Boyles, R. A. (1991) and Dovich, R. A. (1991) for more on this subject.

[40] For non-normal distributions, see the pages that follow.

Handling Non-Normal and Multivariate Distributions[41]

Although the normal distribution is useful in describing and analyzing a wide variety of processes, it cannot be used for all processes. Some processes are inherently non-normal, and their deviations from normality are such that using the normal distribution as an approximation can lead to erroneous decisions. Other processes have multiple characteristics that are interrelated and should be modeled as a multivariate distribution.

Of the indices described above, C_p, P_p, CR, and PR are robust with respect to non-normality. This is not true for C_{pk} and P_{pk}.

Relationship of Indices and Proportion Nonconforming

Although many individuals use the C_{pk} and P_{pk} indices as scalar-less (unit-less) metrics, there is a direct relationship between each index and the related process parameter of proportion nonconforming (or ppm). Assuming that $C_p > 1$, the capability index relationship is given by:

$$proportion\,nonconforming = 1 - \int_{-\infty}^{z_c} e^{-\left(\frac{x}{2}\right)^2} dx$$

where $z_c = 3C_{pk}$ and

$$C_{pk} = \min\{CPU, CPL\}$$

Similarly, P_{pk} is related to the performance proportion nonconformance through:

$$z_p = 3P_{pk}$$

With this understanding of C_{pk} and P_{pk}, indices for non-normal distributions can be developed with the same relationships between the index and the process proportion nonconforming.

The determination of these indices for non-normal distributions requires extensive tables or the use of iterative approximation techniques. They are rarely calculated without the assistance of a computer program.

Non-Normal Distributions Using Transformations

One approach is to transform the non-normal form to one that is (near) normal. The specifications are also transformed using the same

41 As discussed in Chapter II, Section A, process analysis requires that the data have been collected using measurement system(s) that are consistent with the process and have acceptable measurement system characteristics.

parameters. The C_{pk}, and P_{pk} indices are then determined in the transformed space using standard calculations based on the normal distribution.

Two general transformation approaches which have gained support are:

- Box-Cox Transformations

  The methods of analysis of designed experiments are "appropriate and efficient when the models are (*a*) *structurally adequate*, and the (supposedly independent) errors (*b*) have *constant variance* and (*c*) are *normally distributed*.[42]" Box and Cox (1964) discussed a transformation which reasonably satisfies all three of these requirements. This transformation is given by:

  $$w = x^{\lambda}$$

  where $-5 \le \lambda \le 5$

  and $\lambda = 0$ for the natural log transformation

  $\lambda = 0.5$ for the square root transformation

  Although this transformation was developed with the focus of the analysis of designed experiments, it has found an application in the transformation of process data to normality.

- Johnson Transformations

  In 1949, Norman L. Johnson developed a system of transformations which yields approximate normality.[43] This system is given by:

S_B	$w = log\left(\frac{x}{(1-x)}\right)$	Bounded
S_L	$w = log(x)$	Log Normal
S_U	$w = sinh^{-1}(x) = log\left(x + 1 + \sqrt{y^2 + 1}\right)$	Unbounded

As in the case of the Pearson Family of distributions (see below), this system of curves encompasses all the possible unimodal distributional forms; i.e., it covers the entire feasible *skewness-kurtosis* plane. It also contains as a boundary form the familiar lognormal distribution. However, in the general case, the Johnson curves are four parameter functions.

42 Box, G. E. P., Hunter, W. G., and Hunter, J. S., *Statistics for Experimenters*, John Wiley and Sons, New York, 1978, pg.239.

43 See Johnson (1949).

Non-Normal Distributions Using Non-Normal Forms

Non-normal forms model the process distribution and then determine the proportion nonconforming, i.e., the area of the non-normal distribution outside the specifications.

A common approach to the modeling of the non-normal distribution is to use the Pearson Family of Curves. The most appropriate member of this family is determined by the method of matching moments; i.e., the curve with skewness (*SK*) and kurtosis (*KU*) that match that of the sampled distribution is used as a model for the underlying form. As in the case of the Johnson Transformation System (see above), this family of curves encompasses all the possible unimodal distributional forms; i.e., it covers the entire feasible *SK-KU* plane.

To calculate the non-normal equivalent to the P_{pk} index, the non-normal form $(f(x))$ is used to determine the proportion nonconforming, i.e., the area of the non-normal distribution outside the upper and lower specifications:

$$p_L = \int_{-\infty}^{LSL} f(x)\,dx \quad \text{and}$$

$$p_U = \int_{USL}^{\infty} f(x)\,dx.$$

These values are converted to a z value using the inverse standard normal distribution. That is, the z_L and z_U values in the following equations are determined such that:

$$p_L = \int_{-\infty}^{z_L} e^{-\left(\frac{x}{2}\right)^2} dx \quad \text{and}$$

$$p_U = \int_{z_U}^{\infty} e^{-\left(\frac{x}{2}\right)^2} dx$$

Then $$P_{pk} = \frac{\min\{z_L, z_U\}}{3}$$

Although the standard calculation of P_p is a robust estimate, a more exact estimate can be found using the convention that the process spread is defined as the range that includes 99.73% of the distribution (representing the equivalence of a $\pm 3\sigma$ normal distribution spread). The limits of this range are called the "0.135% quantile" ($Q_{0.00135}$) and the "99.865% quantile" ($Q_{0.99865}$). That is, 0.135% of the values of the population are to be found both below $Q_{0.00135}$ and above $Q_{0.99865}$.[44]

[44] For the normal form: $Q_{0.99865} = -Q_{0.00135} = z_{0.99865}s$

$$0.00135 = \int_{-\infty}^{Q_{0.00135}} f(x)\,dx \quad \text{and}$$

$$0.99865 = \int_{Q_{0.99865}}^{\infty} f(x)\,dx.$$

The calculation for P_p then is:

$$P_p = \frac{\textit{specification range}}{\textit{Est } 99.73\%\, \textit{Range}} = \frac{\textit{specification range}}{Q_{0.99865} - Q_{0.00135}}$$

where the non-normal form is used to calculate the quantiles.

The capability index C_p is calculated as above replacing s with $\left(\bar{R}/d_2\right)$.

Because this approach uses the total variation to calculate the proportion nonconforming, there is no analogue of a non-normal C_{pk} available.

An alternate approach to calculating P_{pk} using quantiles is given in some documents by:

$$P_{pk} = \min\left(\frac{USL - \bar{X}}{Q_{0.00865} - \bar{X}}, \frac{\bar{X} - LSL}{\bar{X} - Q_{0.00135}}\right)$$

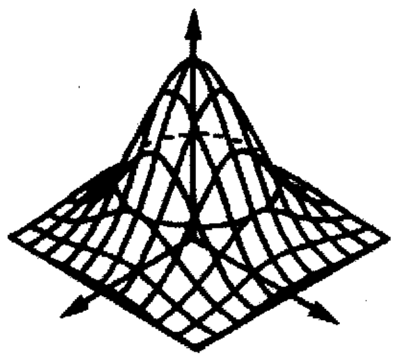
This approach does not tie the P_{pk} index to the proportion nonconforming. That is, different non-normal forms will have the same index for different proportion nonconforming. To properly interpret and compare these indices, the non-normal form as well as the index value should be considered.

Multivariate Distributions

When multiple characteristics are interrelated, the process distribution should be modeled using a multivariate form. The process performance index P_{pk} can be evaluated by first determining the proportion nonconforming, i.e., the area of the multivariate distribution outside the specifications.

For many geometrically dimensioned (GD&T) characteristics, the bivariate normal form is useful in describing the process.

A pair of random variables X and Y have a bivariate normal distribution if and only if their joint probability density is given by



Bivariate Normal Distribution

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{\left(-\frac{z}{2(1-\rho^2)}\right)}$$

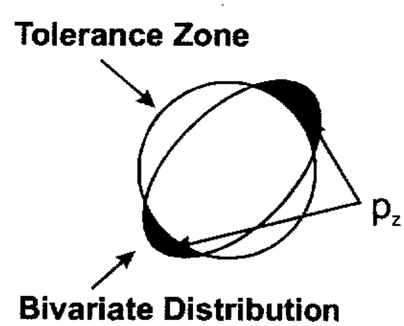
where $z = \left(\frac{x-\mu_x}{\sigma_x}\right)^2 - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \left(\frac{y-\mu_y}{\sigma_y}\right)^2$

$$\rho = \operatorname{cov}(x,y) = \frac{\sigma_{xy}}{\sigma_x\sigma_y}$$

for $-\infty < x < \infty;\ -\infty < y < \infty;\ \textit{where}\ \sigma_x > 0;\ \sigma_y > 0$

To calculate the multivariate equivalent to the P_{pk} index, the multivariate form $(e.g.,\ f(x,y))$ is used to determine the proportion nonconforming, i.e., the volume of the multivariate distribution outside the specification (tolerance) zone. In the bivariate case this would be:

$$p_z = \iint_{\textit{tolerance zone}} f(x,y)\,dx\,dy \quad \text{and}$$



This value is converted to a z value using the inverse standard normal distribution. That is, the z value such that:

$$p_z = \int_{-\infty}^{z} e^{-\left(\frac{x}{2}\right)^2} dx$$

Then $P_{pk} = \frac{z}{3}$

An estimate P_p can be found using:

$$P_p = \frac{\textit{specification area}}{\textit{Est } 99.73\% \textit{ area}}$$

where the multivariate form is used to calculate the estimated 99.73% area.

Because this approach uses the total variation to calculate the proportion nonconforming, there is no analogue of a multivariate C_{pk} available.[45]

[45] See also Bothe (2001) and Wheeler (1995).

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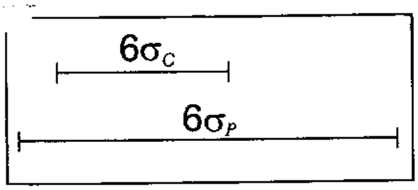
CHAPTER IV - Section C

Suggested Use of Process Measures

The key to effective use of any process measure continues to be the level of understanding of what the measure truly represents. Those in the statistical community who generally oppose how C_{pk} indices are being used, are quick to point out that few "real world" processes completely satisfy all of the conditions, assumptions, and parameters within which C_{pk} has been developed (see Gunter, B. (1989) and Herman, J. T. (1989)). It is the position of this manual that, even when all conditions are met, it is difficult to assess or truly understand a process on the basis of a single index or ratio number, for reasons discussed below.

No single index or ratio should be used to describe a process. It is strongly recommended that all four indices (C_p, C_{pk} and P_p, P_{pk}) be calculated on the same data set. The comparison of the indices among themselves can provide insight to potential process issues and aid in measuring and prioritizing improvement over time. For example, low C_p, C_{pk} values may indicate within-subgroup variability issues, whereas low P_p, P_{pk} may imply overall variability issues.

Graphical analyses should be used in conjunction with the process measures. Examples of such analyses include control charts, plots of process distributions, and loss function graphs.



Additionally, it is helpful to graph the inherent process variation, $6\sigma_C = 6\bar{R}/d_2$ versus total process variation, $6\sigma_P = 6s$, to compare the process "capability" and "performance" and to track improvement. Generally, the size of this gap is an indication of the effect that special causes have on the process. These types of graphical analyses can be done for better process understanding even if process indices are not used.

Process measures should be used with the objective of aligning the "Voice of the Process" to the "Voice of the Customer".[46]

All capability and performance assessments should be confined to single process characteristics. It is never appropriate to combine or average the capability or performance results for several processes into one index.[47]

[46] See Figure I.1.
[47] Methods for addressing multivariate processes are addressed in Chapter IV, Section B.

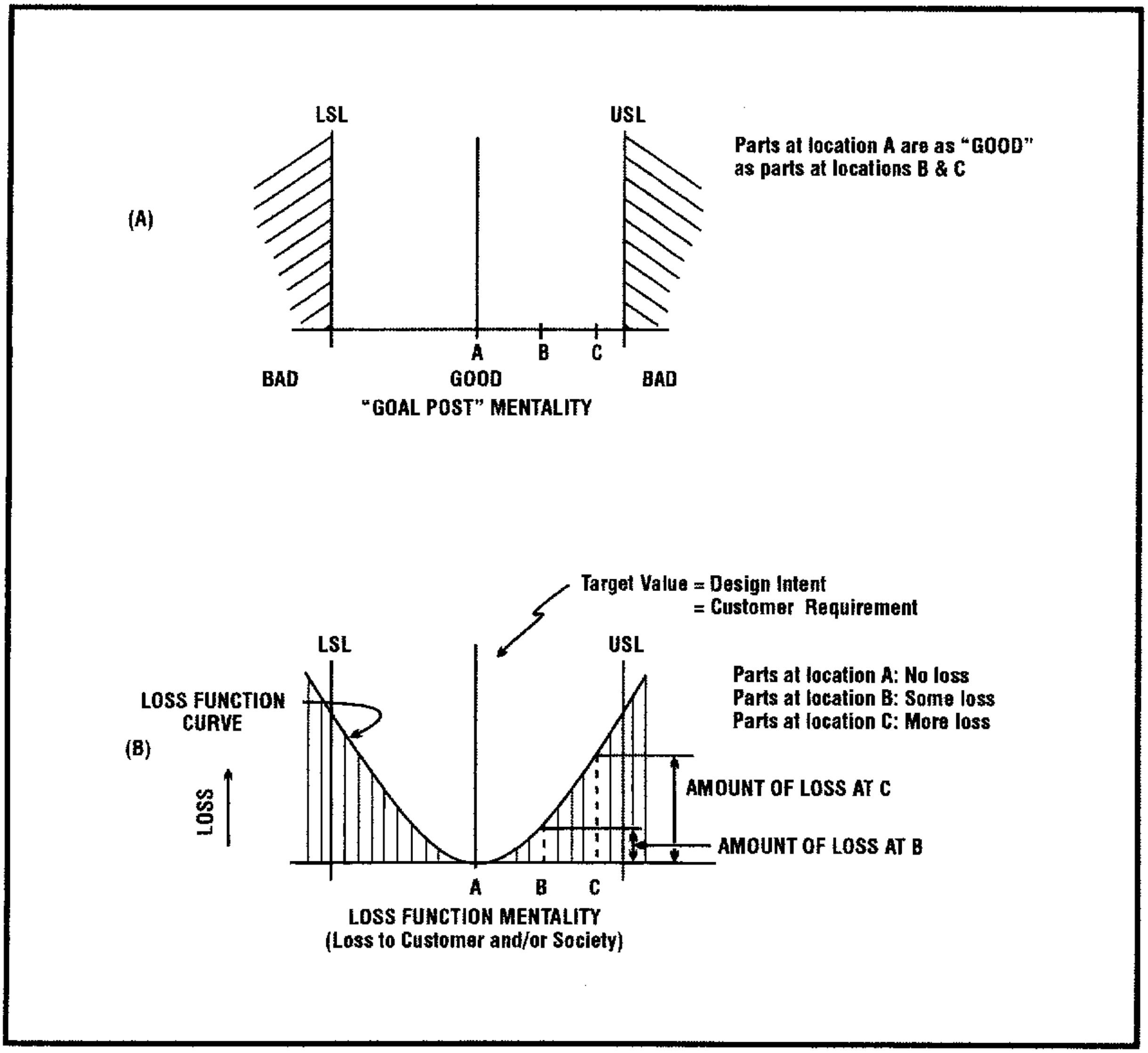


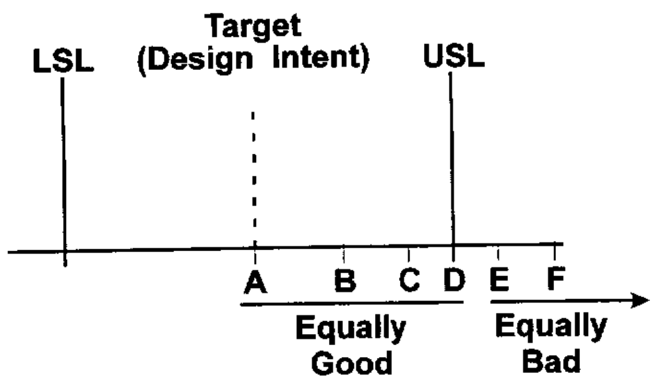
Figure IV.5: "Goal Post" vs. Loss Function

The Loss Function Concept

The driving force behind the use of capability indices (and other process measures) has been the desire to produce all parts within customer specifications. The underlying concept motivating this desire is that all parts within specification, regardless of where they are located within the specification range, are equally "good" (acceptable), and all parts beyond specifications, regardless of how far beyond specifications they may be, are equally "bad" (unacceptable). Quality professionals sometimes refer to this concept as "Goal Post" mentality (see Figure IV.5(A)).

Although this mental model (good/bad) has been extensively used in the past, it is suggested that a more useful model (i.e., one that is a lot closer to the behavior of the real world), is illustrated in Figure IV.5(B). In general, this model is a quadratic form and uses the principle that an increasing loss is incurred by the customer or society the further a particular characteristic gets from the specification target. Implicit in this concept, referred to as the loss function, is the presumption that the design intent (specification target) is aligned with the customer's requirement.

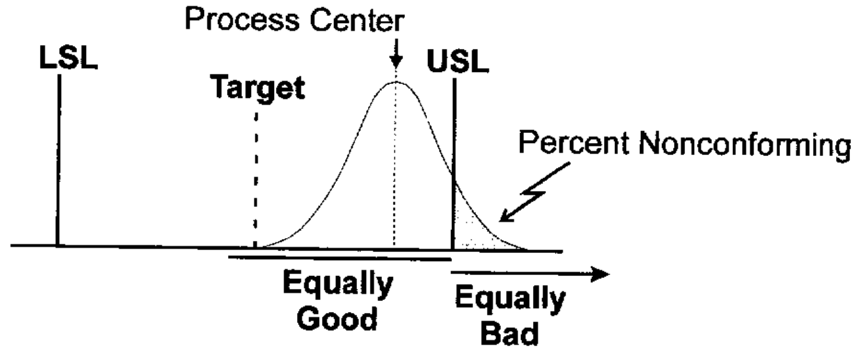
The first step in managing variation is to understand how much variation is acceptable; i.e., how much deviation from a target or nominal value is allowable. Traditionally the value judgment of "acceptable" and "allowable" is based on the design engineer's understanding of the functional requirements and the physics of the design and usage environment (engineering subject matter knowledge), tempered by the economic constraints of the production process. The results of this part of the design process are reflected in the engineering specifications (tolerances).



But what do the specifications mean? Ideally, all characteristics of a design should be equal to the design intent – the target value that would yield perfect results. But variation exists. So what is the difference to the customer between two different parts, one with a characteristic on target and one having the same characteristic off target but within specification?

A common approach can be described using the "Goal Post" analogy. In many sports (e.g., football, soccer, hockey, basketball) a field goal is awarded if the ball passes through the goal posts (or hoop in basketball). It doesn't make a difference if the ball or puck enters dead center or just slips in. The score awarded is the same.

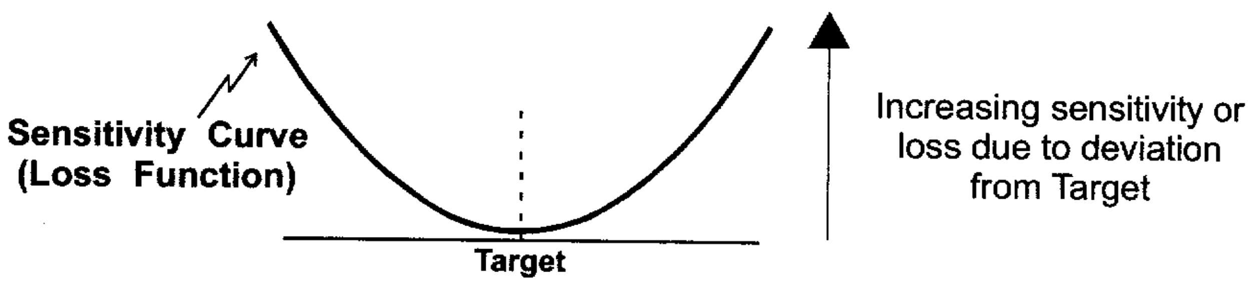
In manufacturing processes this means that everything within the specification limits is considered equally good, and everything outside is equally bad.



This approach may be valid for discrete characteristics (e.g., the part has a clearance hole or not), but when dealing with characteristics with a continuous response, this approach does not reflect how the customer reacts to different levels of the output.

Without considering the specifications, it is possible to determine the customer's sensitivity to deviations from the target (design intent). See Goble, et al (1981). As a characteristic deviates farther from the target, more customers will be able to "sense" that it is different than the design intent - primarily because it takes more "effort" to use. In many cases a loss (in time, cost, efficiency, etc.) can be associated with each deviation increment. This loss can apply to the individual customer, but it also may extend to the organization, or even to society.

A typical sensitivity curve (loss function) has a quadratic form.



There are two ways to analyze a loss function. It can be compared either to the design intent or functional specifications.

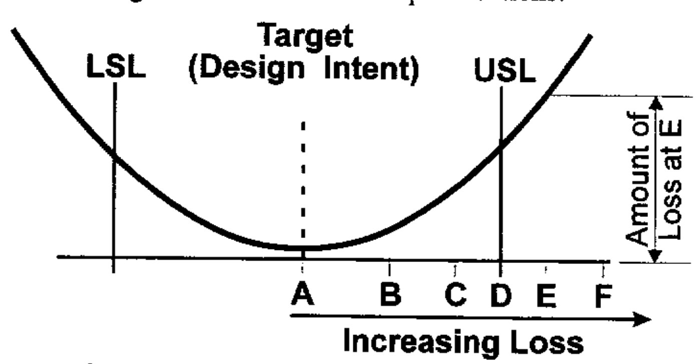


Figure IV.6: Comparison of Loss Function and Specifications

From the customer's perspective, Figure IV.4 shows that there is functionally little difference between a characteristic that is a "nudge" on one side of the specification limit or the other.

A comparison of the loss function to the specifications provides a way to classify characteristics. Figure IV.5 shows that the loss function for Characteristic A is relatively flat within the specification limits. This means that the customer will be insensitive to variation within specification for Characteristic A. Since all characteristics are expected to be within specification, this characteristic satisfies the operational definition for robust.[48]

A characteristic is called ***Robust*** if the customer is insensitive to the characteristic's expected variation.

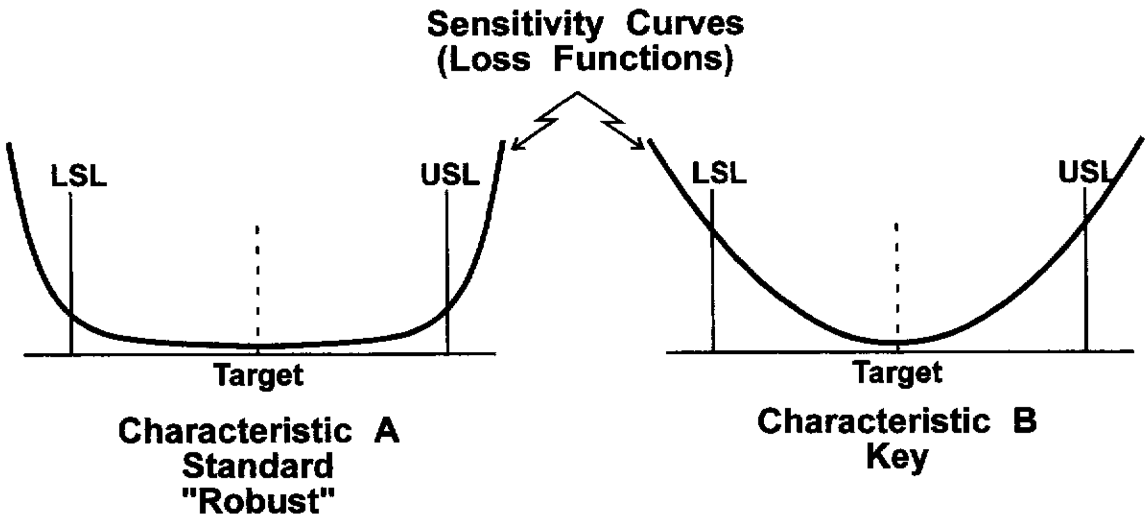


Figure IV.7: Comparison of Loss Functions

[48] Alternative definition: A design is robust if it is tolerant (insensitive) to variation that is expected from the manufacturing, processes, materials and environment.

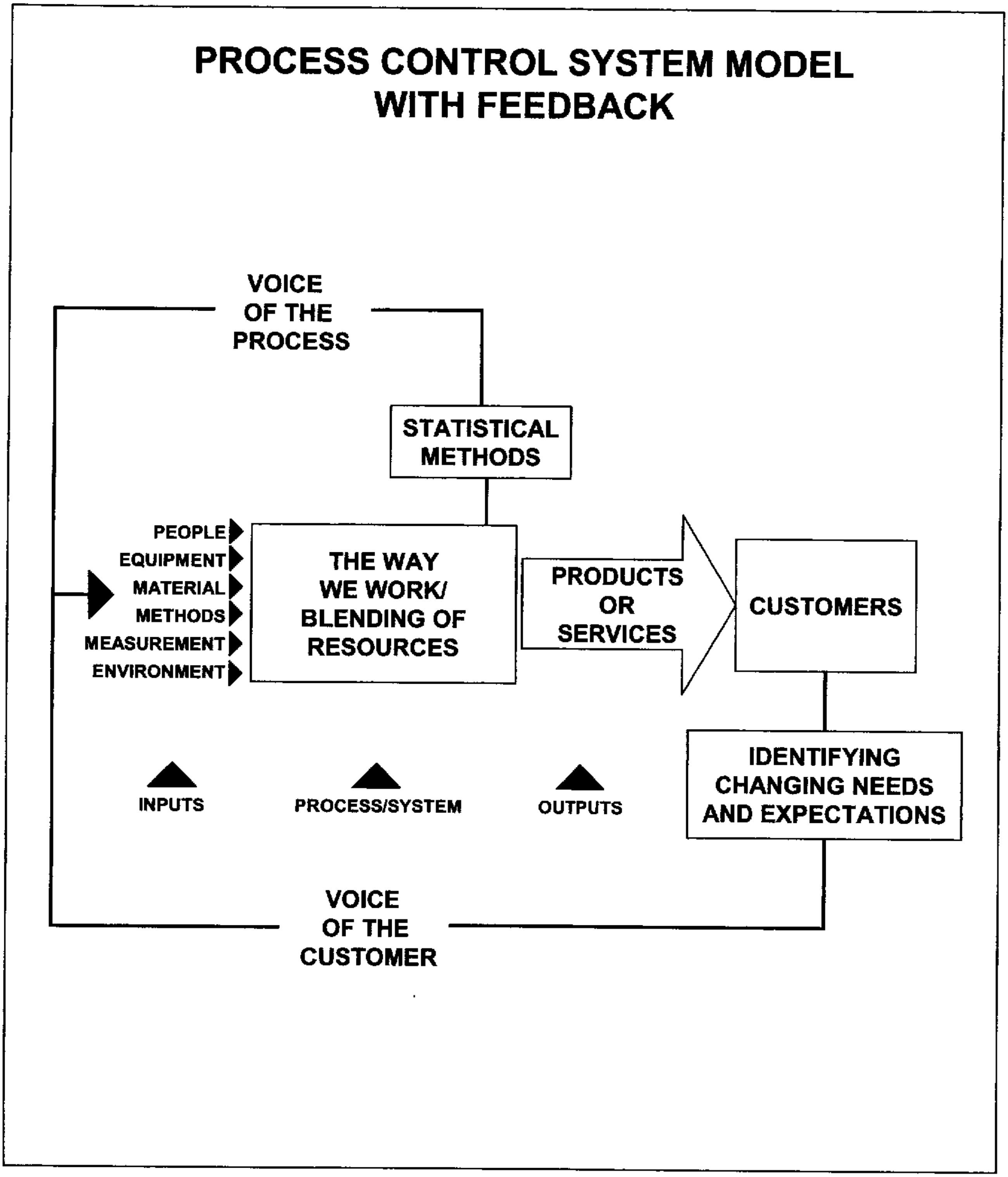
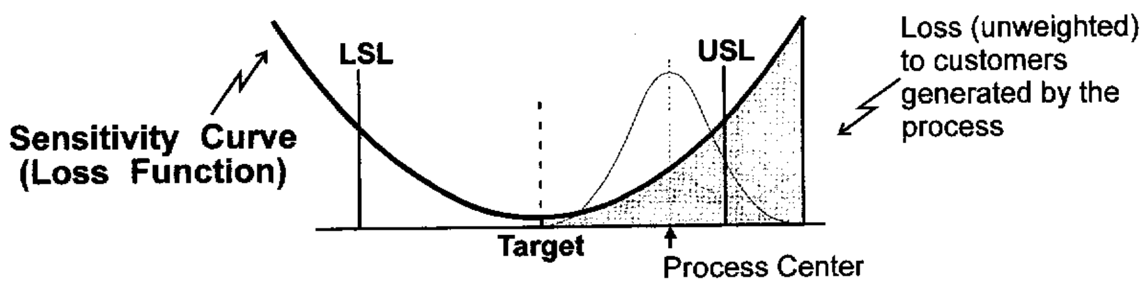


Figure IV.8: A Process Control System

Alignment of Process to Customer Requirements

In Chapter I, Section B (see Figure IV.8), a process control system is described as a feedback system. An output characteristic of such a process can also be expressed graphically in terms of a probability distribution. This distribution might be referred to as the process distribution (see Figure IV.9(a)).



The sensitivity curve also provides direction in the control of the production process. The comparison of the process to the loss function and the specification together shows that the total loss to the customer increases as the process center (average) deviates from the target.

To assess the impact of the process distribution to the customer, a loss function (see Figure IV.9(b)) can be established for the process characteristic. Superimposing the process distribution on the customer requirement loss function curve (see Figure IV.9(c)) shows:

- How well the process center is aligned with the customer target requirement.
- The loss to the customer being generated by this process.

Based upon these observations the following can be concluded:

- In order to minimize customer losses, the process (process center) should be aligned with the customer requirement (specification target).
- It is beneficial to the customer if variation around the target value is continually reduced (see Figure IV.9(e)).

This analysis is sometimes called aligning the "Voice of the Process" with the "Voice of the Customer" (see Scherkenbach, W. W. (1991) for more details).

In the example in Figure IV.9(d), the parts beyond specification account for only 45% of the total loss to the customer. The remaining loss is coming from parts within specification but not at the target.

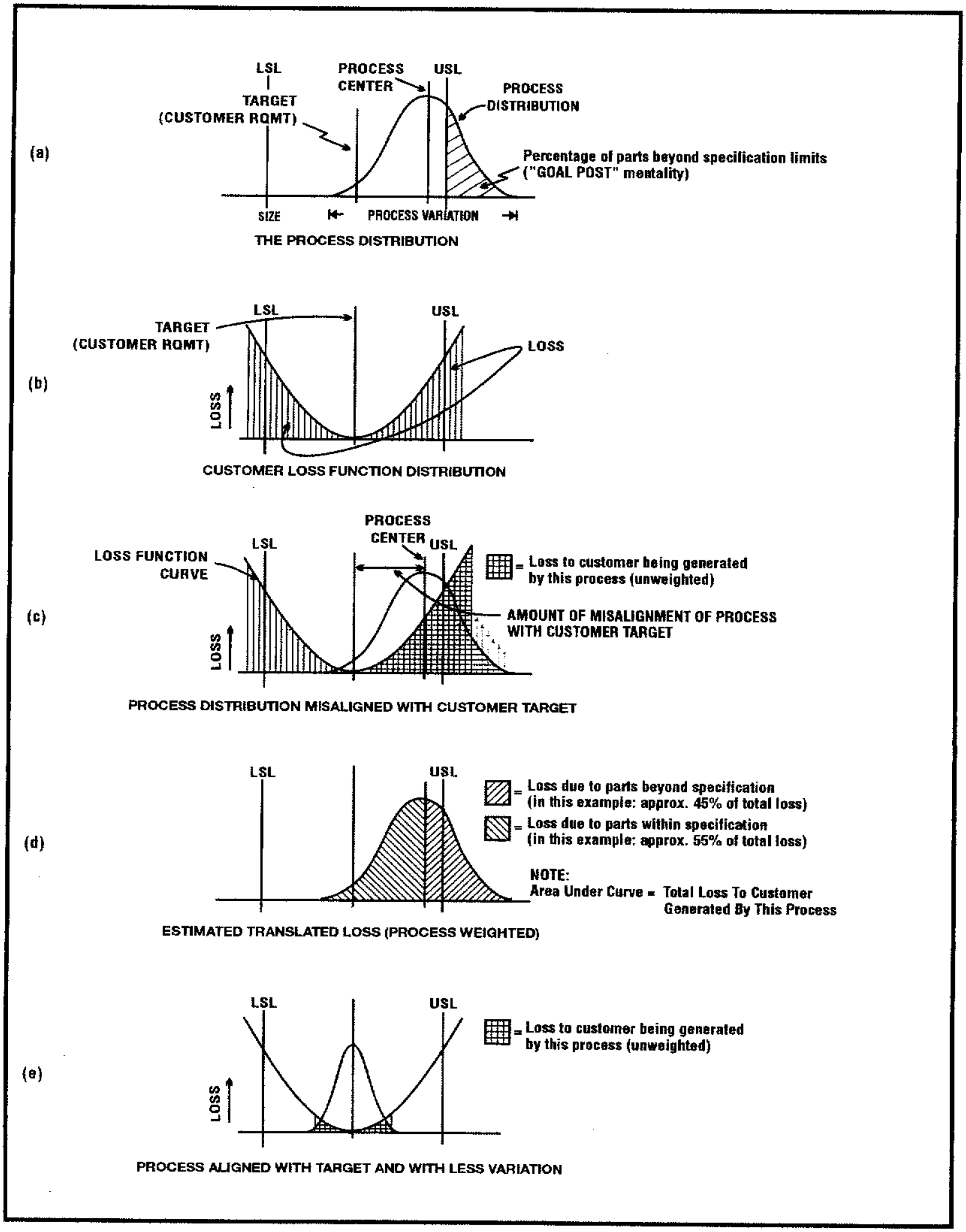


Figure IV.9: Process Alignment to Requirements

This was determined by estimating the total loss: combining the loss generated by the actual distribution of the parts (nonconforming parts) and the loss due to the customer's sensitivity to variation within the specifications. This strongly suggests that the "Goal Post" model, or computing percentage of "bad" parts (parts beyond specifications), in and of itself does not provide a proper appreciation for understanding the effect the process is actually having on the customer.

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APPENDIX A

Some Comments on Sampling

Effects of Subgrouping

Control charts are used to answer questions about a process. In order to have a control chart be useful, it is important that the charts answer the right questions. An $\bar{X}$ chart asks the question, "Is the variation present between subgroup averages more than is expected based on the variation within subgroups?". Therefore, understanding sources of variation within and between subgroups is of paramount importance in understanding the control chart and the process variation. Most variables control charts compare within-subgroup variation to between-subgroup variation, so it is important in interpreting the control charts to form subgroups with an understanding of the possible sources of variation affecting the process results.

Autocorrelated Data

There are generally 3 features to any sampling that is performed when doing SPC:

1. Size: How many parts are selected in the sample?
2. Frequency: How often do we take a sample?
3. Type: Will the sample consist of consecutively selected pieces, randomly selected[49] pieces, or some other structured plan?

Of the 3 features mentioned above, most people are experienced with 1 and 2, but 3 is seldom considered. In fact, sample *type* is not even covered in most control plan templates. The type of sample can have a large impact on the results of SPC charting and should be understood.

Some factors that influence the impact of the sample type have to do with the process itself – they are dependent on the nature of the manufacturing process. One particular phenomenon common with many modern day, high speed, automated processes is known as *autocorrelation*.

The concept of correlation may be familiar to many people. There are many examples of correlation which are part of experience in everyday life (e.g., height/weight) where two features are compared in order to determine if there appears to be a significant relationship between them. As the value of one feature rises, the value of the other feature may rise with it (indicating a positive correlation), or it may fall with it (indicating

[49] It is important to understand the real meaning of "random". In practice, many people think that by blindly selecting pieces that what they are doing is "random" selection. In reality, this may be haphazard sampling or convenience sampling. Selection of a random sample requires specific techniques to ensure that the sample is random. Using haphazard or convenience sampling when random sampling is required can lead to erroneous and biased conclusions.

a negative correlation), or it may act independently of it (zero correlation). The mathematical formula for correlation leads to a value between -1 (negative correlation), through zero (no correlation), to +1 (positive correlation). In order to achieve these results, several samples are taken from the population and the two features of interest are compared within each other. In the production world, different characteristics of the same product/process may be compared.

In autocorrelation, instead of comparing two features within a part, one feature is compared to that same feature on a part produced before it. It may be compared to the part produced immediately prior to it (called a *lag* of 1), or two parts prior (a *lag* of 2), etc.

High-speed, automated processes are often found to exhibit autocorrelation on some characteristics. This is often because there is an underlying predictable special cause variation which is large when compared to the common cause variation. That is, the important process input variables have not had time to vary much in the period of time the sample was taken when compared to the between sample variation. This may be illustrated with examples.

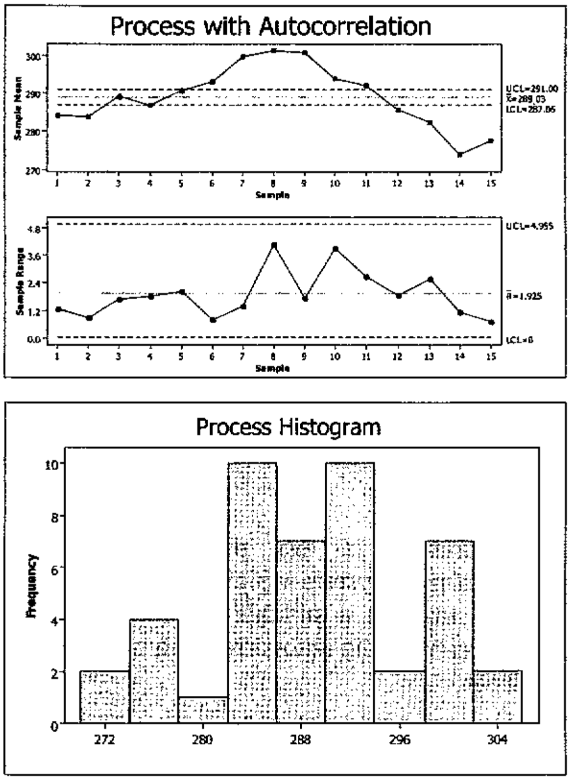
Temperature example:

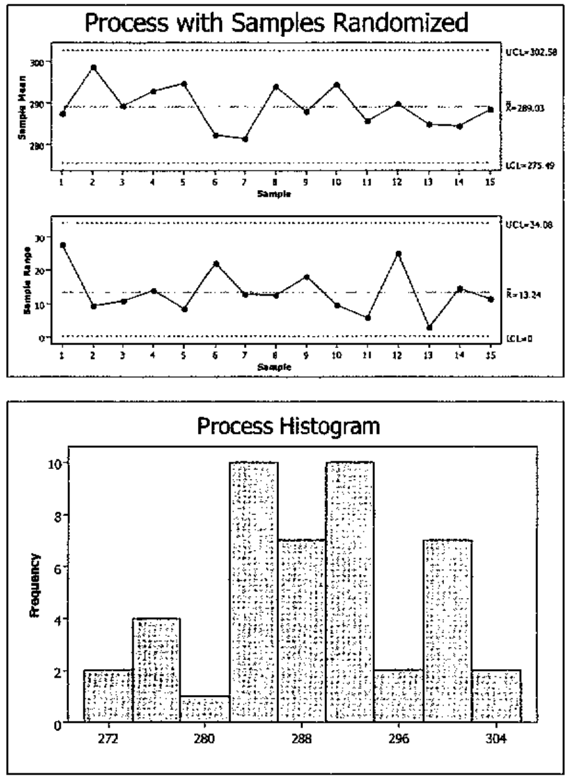
If one were asked to do an $\bar{X}$ *and* R control chart on the temperature of a room (or outside patio), it does not make sense to have a sampling plan that calls for taking 5 consecutive temperature readings – each of the 5 values would be essentially the same as each other. However, an hour later when the next sample is taken, the temperature would likely be different than it was an hour before, yet those 5 readings would again be the same as each other. And so on.

When such a chart is completed, there would likely be some apparent random variation in the $\bar{X}$ chart, but the Range chart would be primarily a stream of zeroes. The average range would then be approximately zero. $\bar{R}$ is used to calculate the control limits on the average, in the formula $\bar{\bar{X}} \pm A_2 \times \bar{R}$, so the control limits would be extremely tight on the grand average of the data, and most points would show as being out of control. This is an extreme example, but it serves to point out what happens when autocorrelation is present and is ignored.

Stamping example:

Data from a coil-fed, progressive die process is typically autocorrelated. If this data were randomized (i.e., coil-fed steel were to be cut into blanks, randomized and then measured), the data would then not be autocorrelated. Yet the final shipped outcome (the total process distribution as indicated by a histogram) would be identical. The underlying cause for the autocorrelation has been broken. Is this practical or feasible to do forever? No, not in this case – but this example does serve to illustrate the possible nature of autocorrelation in a process.





Autocorrelation can lead to misleading conclusions if C_{pk} is calculated while ignoring its effect on the process variation. Since C_{pk} is based on $\overline{R}$ (a within-subgroup estimate of the standard deviation), it is evident in the above example that the C_{pk} will be extremely high, yet it is obvious that there is more variation in the process which has not been captured by C_{pk}.

Identifying Autocorrelation

To discover if a process is autocorrelated, firstly, consider the process inputs in terms of the 6M's.[50] If a process is highly dependent on the operator, it is not likely that the process would be autocorrelated. On the other hand, if the process is highly dependent on raw material and that raw material is a continuous variable (such as a coil of steel used to feed a metal stamping process), autocorrelation within each coil is highly likely. Similarly, for a process which is highly dependent on specific machine related characteristics (such as a stamping press and die combination as affected by lubrication, die temperature, tool condition, etc.). When a process is both material and machine dependent, autocorrelation may be significant.

[50] Man, Material, Method, Machine, Mother Nature (Environment), Measurement System.

Secondly, there are statistical analyses[51] that can be used to determine the actual autocorrelation coefficient and pattern. The methodology of paired sample correlation analysis can be used to compare the current sample to the prior sample, then the next sample to the current sample, etc. When samples from a process are stable and independent, the plotted point will be positioned "randomly" (random from a normal distribution) between the control limits. The plotted points from an autocorrelated process will not vary far from their neighboring sampling points, forming a lazy, wandering pattern.

Ways to Address Autocorrelation

Often, nothing can be done to change an autocorrelated process. Different sampling methods may be called for.

I and MR:

If the within-subgroup variation is less than or equal to the discrimination of the measurement system which is appropriate for the process, an *I and MR* chart may be a suitable method to control the process variation. However, very strong autocorrelation may still display itself in a non-random pattern.

Structured Samples:

The selection of the sampling quantity and frequency should reflect the dominant sources of variation. For example, if the process is material dominant, then the sampling should occur whenever the material changes (e.g., with the change of coils).

Autoregressive Charts:

In cases where the assumption that the sample data are independent is violated, an autoregressive model would be appropriate. See Chapter III.

Structured Charting:

If the source (special cause) of the autocorrelation is predictable, it is possible to control the process by segregating the within-subgroup variation from the between-subgroup variation on separate charts. The Between/Within chart utilizes an *I and MR* chart approach as well as the typical *Range* chart:

- The *Individuals* chart plots the subgroup averages treated as individuals against the control limits based on the *Moving Ranges*.
- The *MR* chart plots the between-subgroup variation using the moving ranges based on the subgroup averages.
- The *Range* (or *Standard Deviation*) chart plots the within-subgroup variation.

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[51] The Durbin-Watson test statistic is one method to determine the degree of autocorrelation and is included in many statistical software packages. See *Biometrika*, 38, pp. 159-178, 1951.

[52] For example, see Appendix A for discussion on autocorrelation.

These would be analyzed using the standard control charting methods to assure that both the common cause (within-subgroup) variation and the cause of the autocorrelation (between-subgroup) remain consistent (see Wheeler (1995)).

Summary

What is important here is considering the concept of autocorrelation and the ability to recognize it in a process, then understanding its possible impact on statistical results.

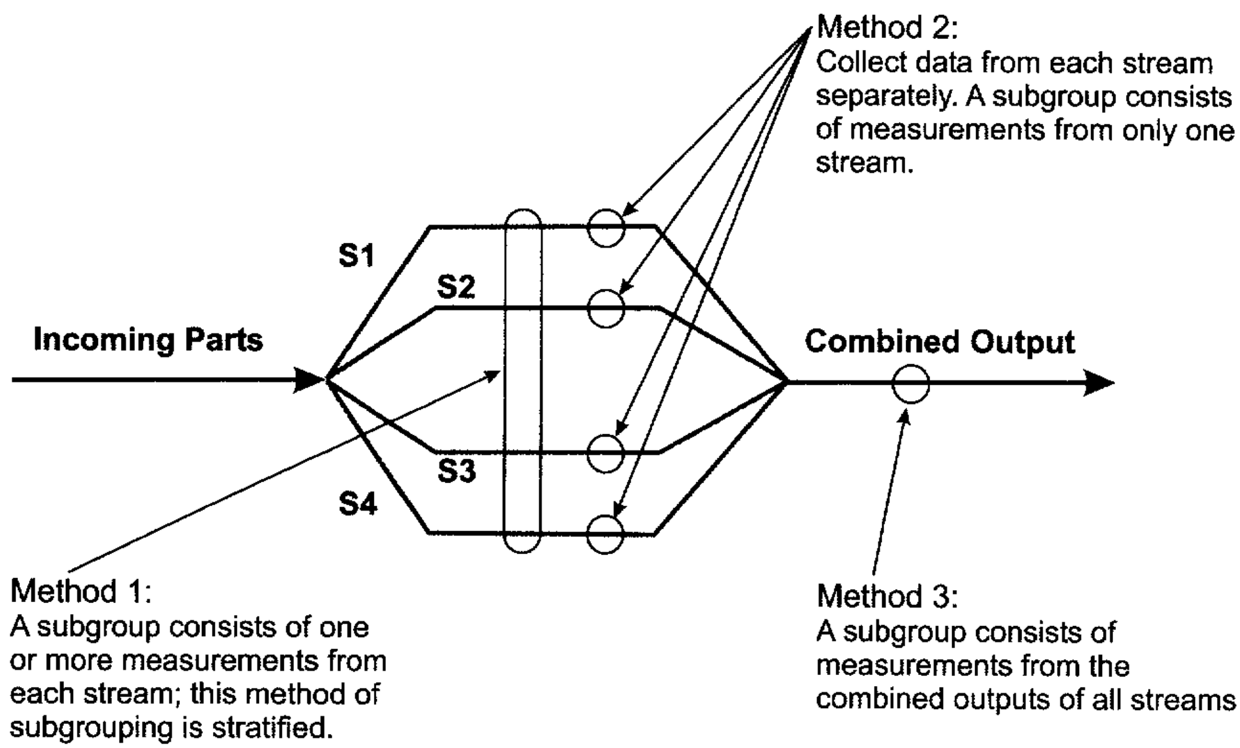
This discussion of autocorrelation is intended only to raise awareness that such a phenomenon exists, how to recognize it, and that its effects, if not recognized or understood, can be quite harmful to otherwise good SPC practices. If the reader should suspect autocorrelation in a process, then a statistician should be consulted.

It is important to understand the real meaning of "random". In practice, many people think that by blindly selecting pieces here and there that what they are doing is "random" selection. In reality, this may be haphazard sampling or convenience sampling (see Glossary). Selection of a random sample requires specific techniques (see a statistical reference book). Using haphazard or convenience sampling when random sampling is required can lead to biased and therefore erroneous conclusions.

Multiple Stream Process Example

Consider the following example: A production process consists of four parallel operations. It is suggested that variation in process output should be studied with control charts, so a decision needs to be made on how to collect the data for the charts. There is a variety of possible sampling schemes that could be considered. Parts could be taken from each stream to form a subgroup, or parts from only one stream could be included in the same subgroup, or subgroups could be formed by taking parts from the combined stream of output without regard to their source. The numerical example below provides an example of possible results obtained using these three methods.

Methods to collect data from the output of a multiple stream (spindle) production process



Every hour a 16 part sample is collected by taking the parts from four consecutive cycles from each stream.

The following is an example of the data.

	CYCLE OF THE MACHINE			
SAMPLE #	**A**	**B**	**C**	**D**
Stream S1	17	18	18	20
Stream S2	12	15	12	12
Stream S3	9	10	9	12
Stream S4	10	11	12	12

There are three sources of variation captured in the data. Cycle-to-Cycle variation is captured by different columns in the array, stream-to-stream variation is captured by the rows of the array, and hour-to-hour variation is captured by different samples of 16 parts.

One subgrouping scheme would be to plot the average and range of each column of each array of data. Using this subgrouping scheme, stream-to-stream variation would be contained within each subgroup. Hour-to-hour variation and cycle-to-cycle variation would contribute to differences between subgroups. Another possible subgrouping scheme would be to plot the average and range of each row of each array of data. With this subgrouping scheme, cycle-to-cycle variation would be contained within each subgroup and hour-to-hour and stream-to-stream variation would contribute to differences between subgroups.

Subgrouping by Column **Subgrouping by Row**

#1	A	B	C	D	X	R
S1	9	10	9	12	10.0	3
S2	12	15	12	12	12.8	3
S3	17	18	18	20	18.3	3
S4	10	11	12	12	11.3	2
X	12.0	13.5	12.8	14.0		
R	8	8	9	8		

#2	A	B	C	D	X	R
S1	9	12	12	11	11.0	3
S2	11	13	10	11	11.3	3
S3	15	17	17	17	16.5	2
S4	12	15	12	11	12.5	4
X	11.8	14.3	12.8	12.5		
R	6	5	7	6		

#3	A	B	C	D	X	R
S1	10	12	11	10	10.8	2
S2	11	12	12	11	11.5	1
S3	16	16	20	16	17.0	4
S4	10	11	12	11	11.0	2
X	11.8	12.8	13.8	12.0		
R	6	5	9	6		

#4	A	B	C	D	X	R
S1	7	6	6	8	6.8	2
S2	8	8	8	9	8.3	1
S3	14	15	15	13	14.3	2
S4	6	7	7	7	6.8	1
X	8.8	9.0	9.0	9.3		
[illegible]	8	[illegible]	[illegible]	[illegible]		

#5	A	B	C	D	X	R
S1	11	11	8	10	10.0	3
S2	13	15	13	15	14.0	2
S3	17	18	19	16	17.5	3
S4	11	12	13	11	11.8	2
X	13.0	14.0	[illegible]	13.0		
R	[illegible]	7	[illegible]	6		

#6	A	B	C	D	X	R
S1	8	10	7	6	7.8	4
S2	9	10	9	9	9.3	1
S3	15	15	14	15	14.8	1
S4	6	7	5	6	6.0	2
X	9.5	10.5	8.8	9.0		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		

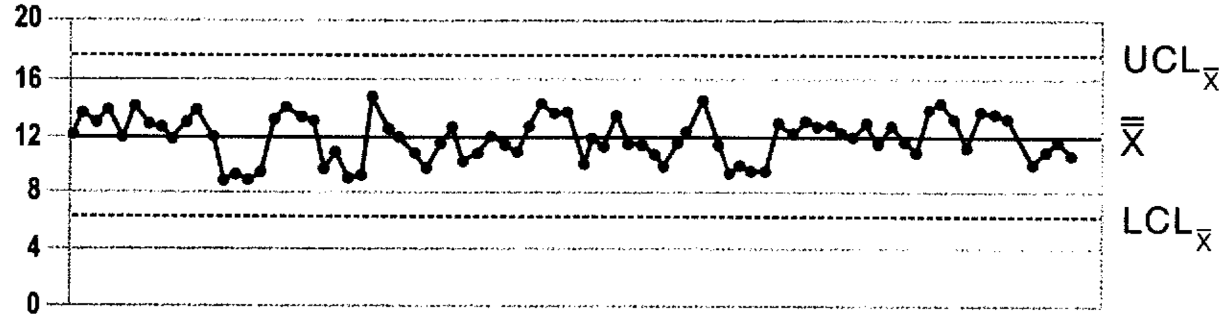
Data from 20 consecutive hours are used to construct control charts with each subgrouping method.

Method 1: Subgrouping by column (Cycle)

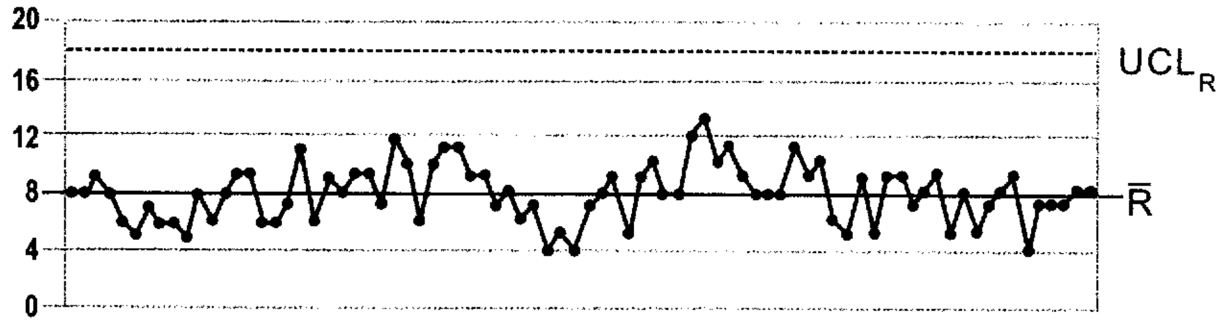
This subgrouping scheme yields 80 subgroups of size n = 4. The grand average is 11.76 units. The average range is 7.85. The control limits for the $\bar{X}$ chart are 17.48 and 6.04 units, and the upper control limit for the range chart is 17.91 units.

A review of the Range chart indicates that the within-subgroup variation appears to be stable using this method.

X-Bar Chart for Data Subgrouped by Column (Cycle)



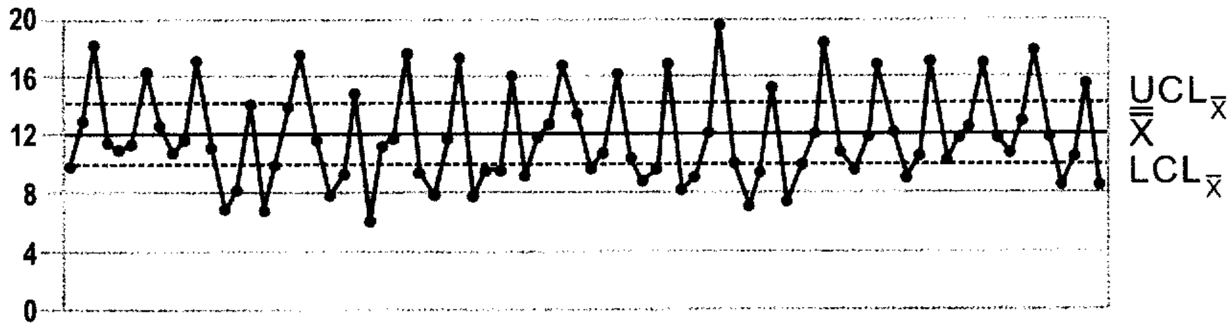
Range Chart for Data Subgrouped by Column (Cycle)



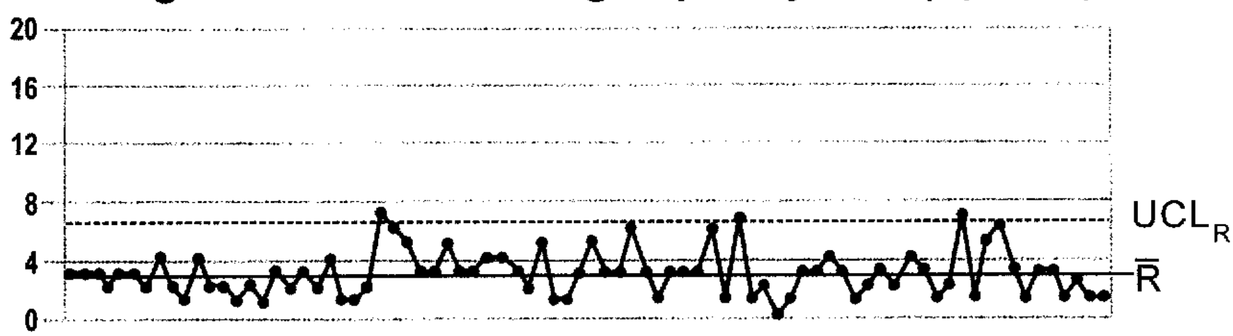
Method 2: Subgrouping by Row

The second subgrouping scheme yields 80 subgroups of size n = 4. The average range is 2.84 units. The control limits for the $\overline{X}$ chart are 13.83 and 9.70 units, and the upper control limit for the range chart is 6.46 units. The control charts for this subgrouping scheme are shown below.

X-Bar Chart for Data Subgrouped by Row (Spindle)



Range Chart for Data Subgrouped by Row (Spindle)



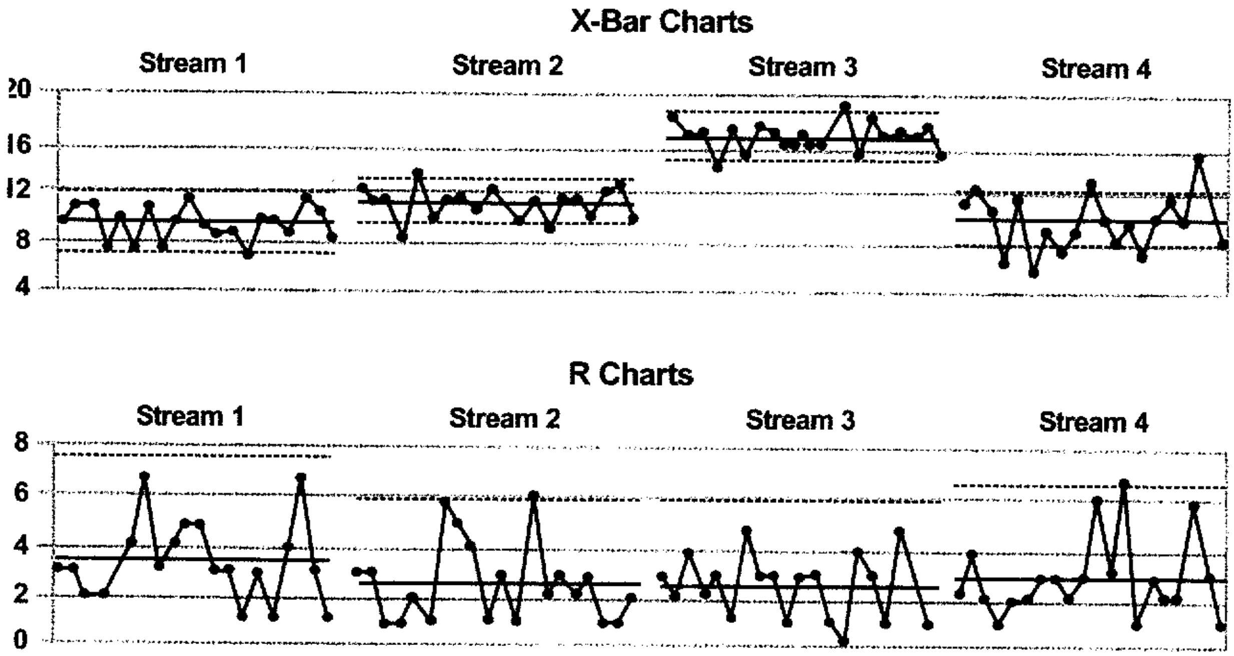
The control charts for the different subgrouping schemes are very different even though they are derived from the same data. The $\overline{X}$ chart for data subgrouped by row shows a pattern: All of the points corresponding to spindle 3 are noticeably higher than those from the other streams. The first $\overline{X}$ chart does not reveal the stream-to-stream differences because readings from each stream are averaged to obtain each $\overline{X}$ value.

By grouping the data differently, the charts address different questions.

For the first set of charts, stream-to-stream variation is used as a basis of comparison. The R chart checks to see that stream-to-stream variation is stable over time and the $\overline{X}$ chart compares cycle-to-cycle and hour-to-hour with stream-to-stream variation.

The second set of charts use cycle to cycle variation as a basis for comparison. The R chart checks to see that the cycle-to-cycle variation is stable over time and the $\overline{X}$ chart compares stream-to-stream variation and hour-to-hour variation with the base level of variation established by the ranges; i.e., cycle-to-cycle variation. The second set of charts identify that a special cause is affecting the process; i.e., the third stream is different from the other streams. Since the stream to stream differences are so large, the control limits in the first set of charts are much wider than the second set.

With the second subgrouping method, the data could be used to create four separate sets of control charts from the data, one for each stream.

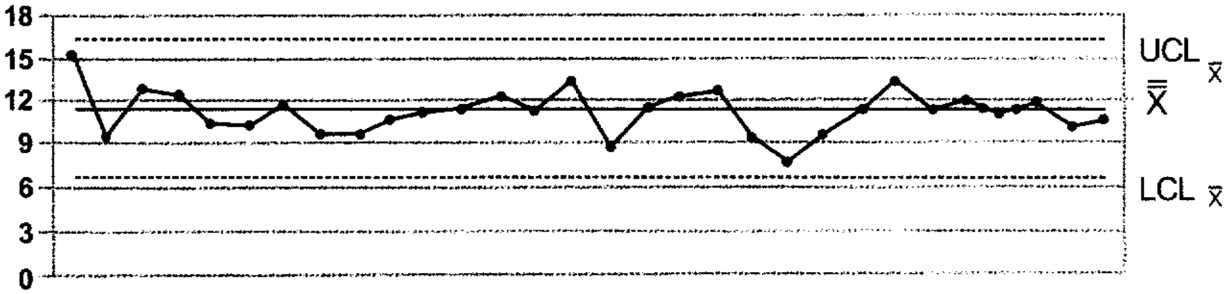


This comparison of the charts shows that the average of the third stream is higher than the others and the individual processes are out of control. The base level of variation used for study of the results from each stream is cycle-to-cycle variation as reflected in the range. For each stream the effects of hour-to-hour variation are shown on the $\overline{X}$ charts. By plotting the charts using the same scale, the level and variation for each stream can be compared.

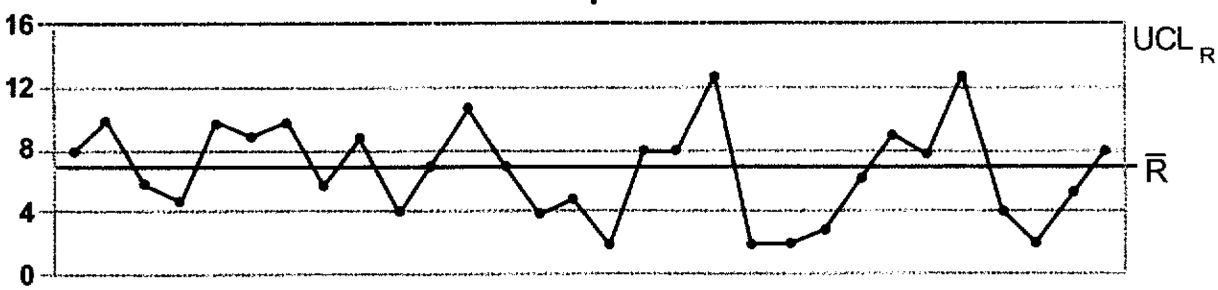
Method 3:

The third method of sampling would be to sample the parts from the combined output from all four streams. This method gives some insight into the variation that is sent to the next process but the parts can no longer be differentiated by production stream. Provided the parts in the combined stream are mixed, the ranges reflect a mixture of stream-to-stream and cycle-to-cycle variation. The $\overline{X}$ values contain, in addition, hour-to-hour variation. If the hour to hour contribution to variation is large enough, that contribution will be seen as out-of-control points on the $\overline{X}$ chart.

Combined Output X-Bar Chart



Combined Output R Chart



The R chart checks to see if stream-to-stream and cycle-to-cycle variation is consistent over time. The $\overline{X}$ chart answers the question, "Is the variation in $\overline{X}$ values what would be expected if cycle-to-cycle and stream-to-stream variation were the only kinds of variation present in the process, or, is there additional change hour to hour?"

As a general rule, the variation that is represented within subgroups should be the kind of variation that is believed to be the least significant or least interesting as a subject for current study. In all cases, a method of subgrouping should be used that will allow questions about the effects of potential sources of variation to be answered.

Effects of Sample Size on Indices

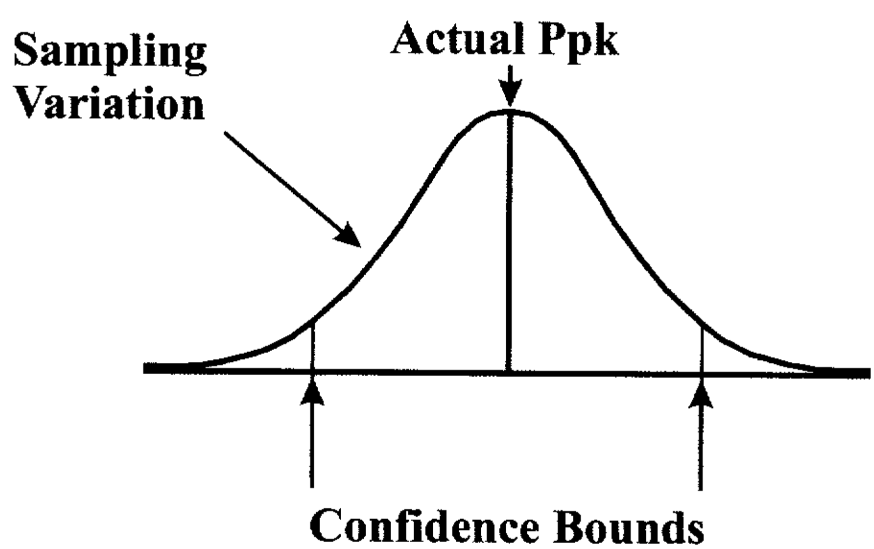
A common scenario when evaluating a process using indices is that the results of one sample may seemingly contradict the results of a second sample. This is especially prevalent with new processes where the initial sample taken to qualify the process for production has index values that meet or exceed the customer requirements but a subsequent sample taken during normal production has indices that fall short of the requirements.

The reasons for this are varied:

- The process has changed from the initial sampling to the full production sampling – e.g., the initial sampling may have been using different material, setups, procedures, etc.
- The initial sampling did not include all the possible sources of variation which are affecting the production process. This is a real possibility if the initial sample size is small.
- The actual process index is close to the target index and sampling variation is causing the difference in conclusions.

The first two reasons relate to the understanding of the sources of variation acting on the process and are discussed in Chapter I.

The third reason deals with the sampling variation inherent in any sampling scheme (see also Chapter I, Section G). Unless the sample includes all the output of the process, there will be sampling variation[53] when calculating a statistic (in this case an index) of the process distribution.

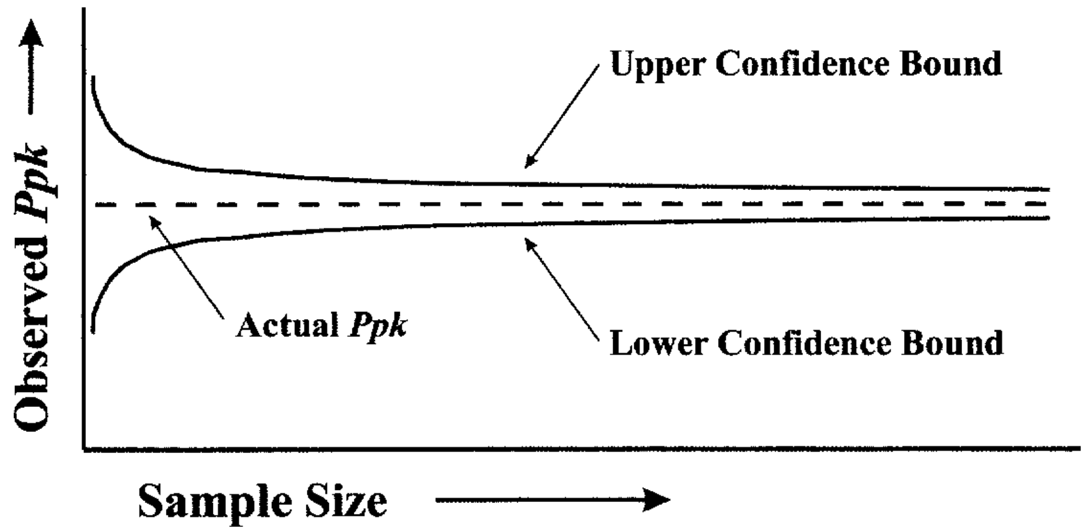


Using the sampling distribution (the distribution of the statistic (index)), it is possible to calculate confidence bounds for the index. These values

[53] Note: Although the actual sampling distribution of the indices will generally be non-normal, this discussion will use a symmetric distribution as an example.

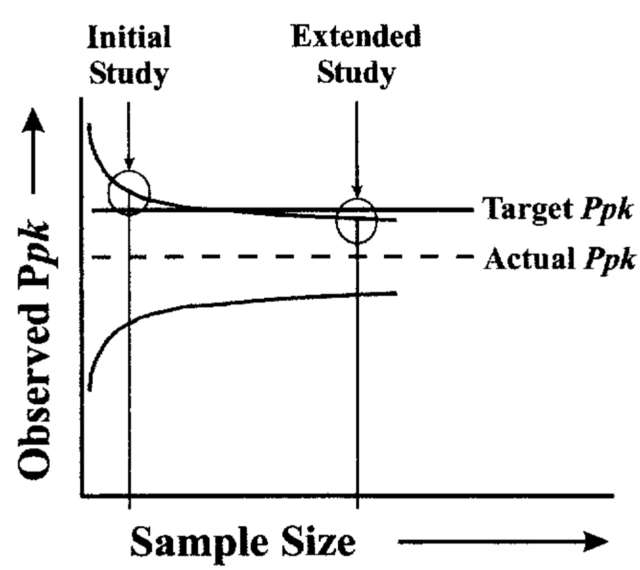
can then be used to make a decision about the process (e.g., is it acceptable or not).

Using a common alpha risk level of .05 as an example, the 95% confidence bounds will identify the range of possible values that will contain the actual (and unknown) value 95% of the time. That is, if the sampling was identically repeated 100 times, the same decision (acceptable or unacceptable) on the process would be made 95 times.

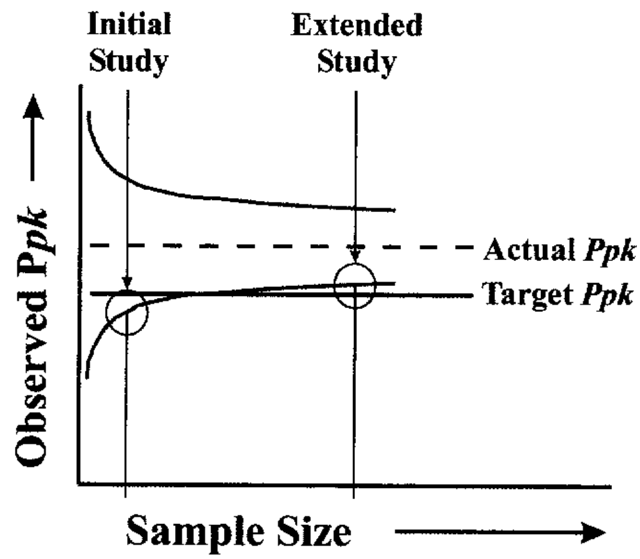


The width of the sampling distribution is a function of the sample size. The larger the sample size the "tighter" the sampling distribution. It is this attribute of the sampling distribution that can lead to seemingly contradictory conclusions.

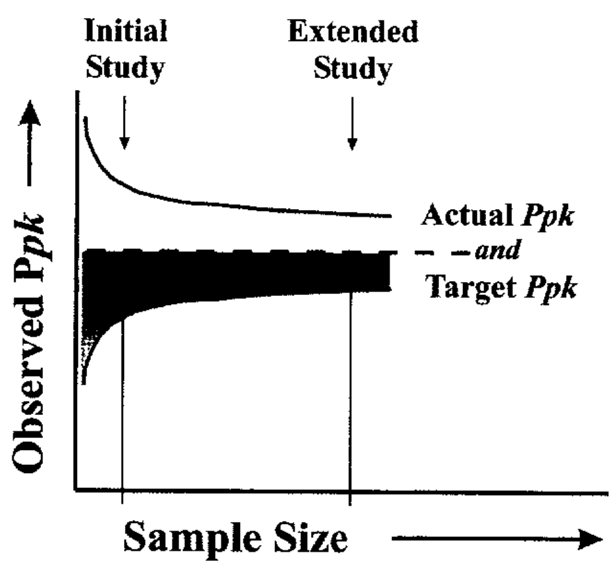
For example, when evaluating new processes the initial sample is usually small due to the availability of raw materials/parts. Once the process is in production, this constraint is not present.

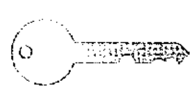


When the actual index is close to the target index then the differences in sampling variation can lead to seemingly contradictory conclusions even if there are no changes in the process and both samples encompass the same sources of variation.



In the case where the actual index (unknown) is exactly equal to the target index then, regardless of the sample size, the probability of calling the process acceptable is only 50%. In other words, the calculated index will be greater or equal to the target index only half the time.





The sample size used in a process study and how close the actual index is to the target index has a significant impact on the validity of any *predictive* decision made about the process.

APPENDIX B

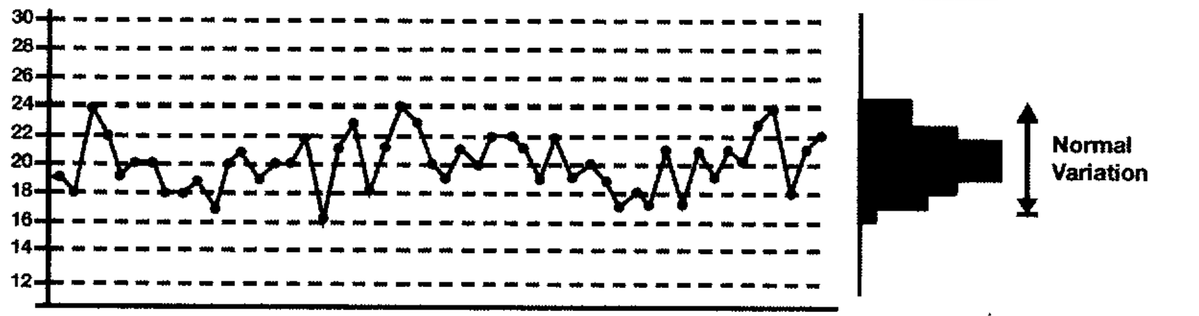
Some Comments on Special Causes

Over-Adjustment

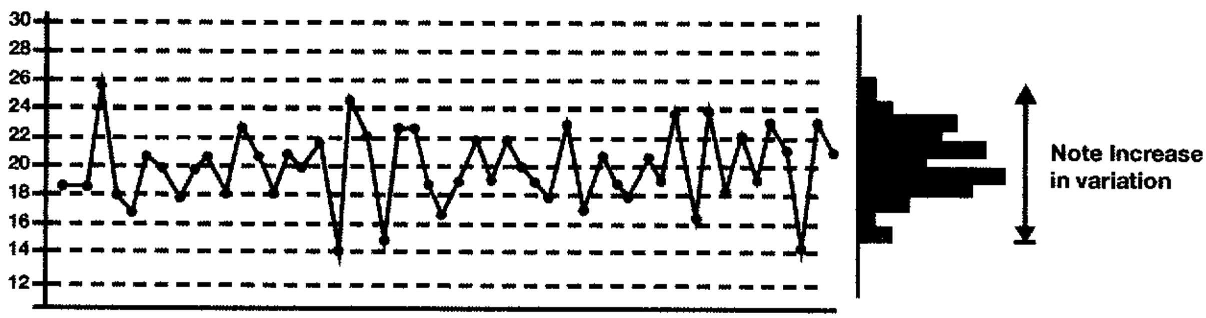
Over-adjustment is the practice of treating each deviation from the target as if it were the result of the action of a special cause of variation in the process.

If a stable process is adjusted on the basis of each measurement made, then the adjustment becomes an additional source of variation. The following examples demonstrate this concept. The first graph shows the variation in results with no adjustment. The second graph shows the variation in results when an adjustment is made to the process to compensate for each deviation from the target. The third graph shows variation in results when adjustments are made to compensate only when the last result was more than one unit from the target. This third case is an example of compensation to stay within a set of specifications. Each method of adjustment increases the variation in the output, since the variation without adjustment is stable (see Deming (1989), Chapter 11).

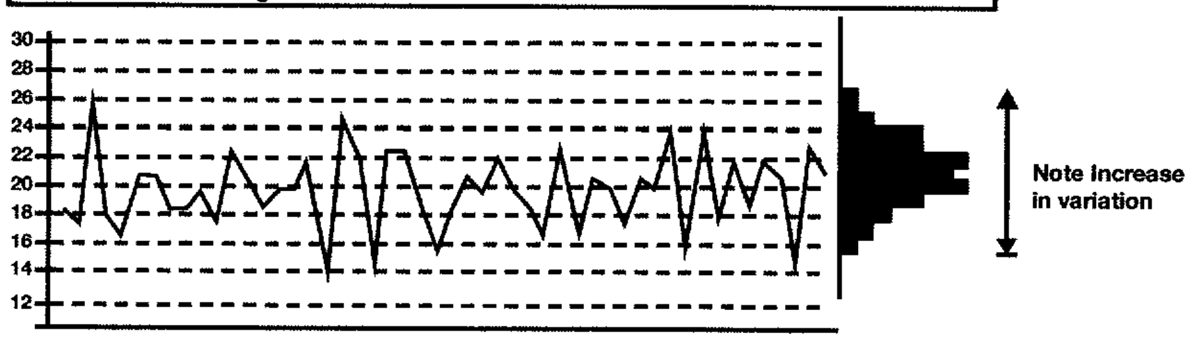
Results with no adjustment



Results with adjustment to compensate for last deviation from target



***Results with adjustment to compensate for last deviation from target if deviation was greater than 1**

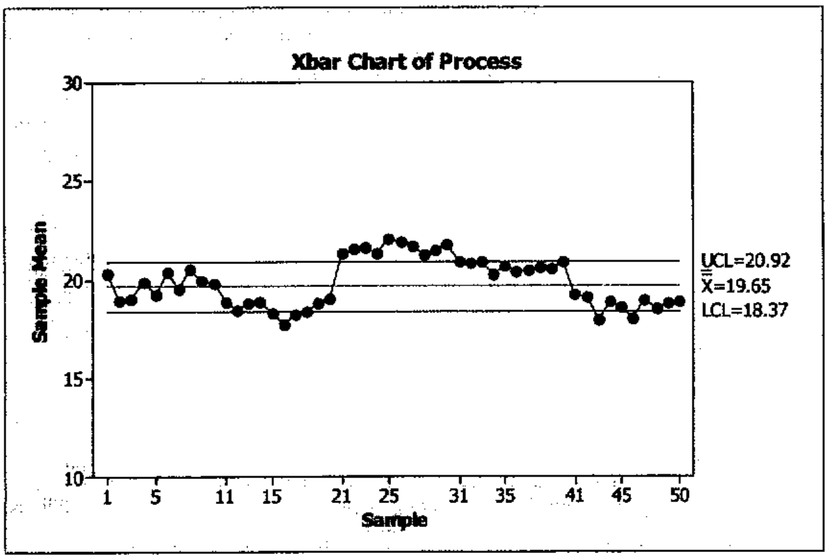


NOTE: These charts assume the measurement system has been assessed and is appropriate.

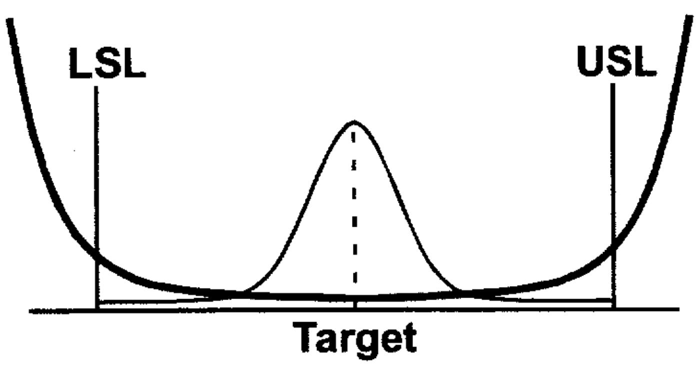
Time Dependent Processes

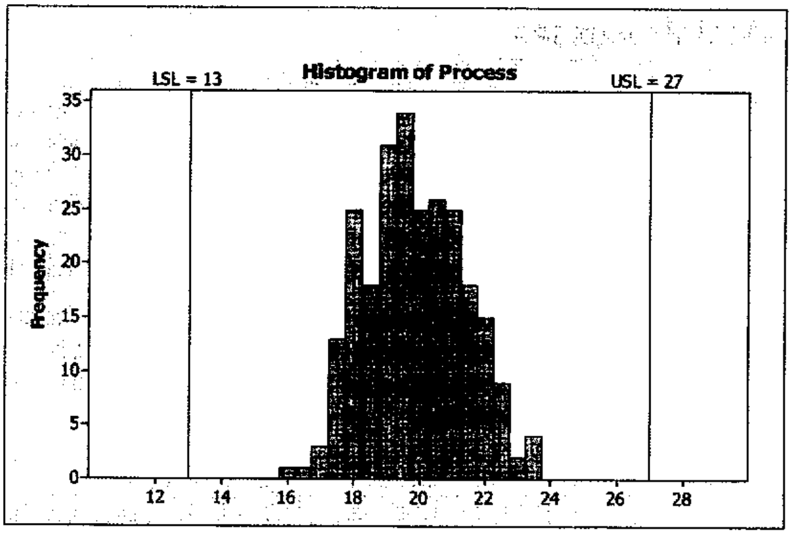
Case 3 processes (see Chapter I, Section E) are usually difficult to fit into the classical control chart model. Few processes of these types remain strictly stable over time. Because the within-subgroup variation is usually small, minor fluctuations in process location or dispersion may cause a process to be out of statistical control, when, in fact, the condition has a minimal practical effect on product quality and the customer.

For example, consider a process that has a fairly constant dispersion, but has small, random location changes. When the control limits were established based on the first 25 samples, numerous points out of control were revealed.

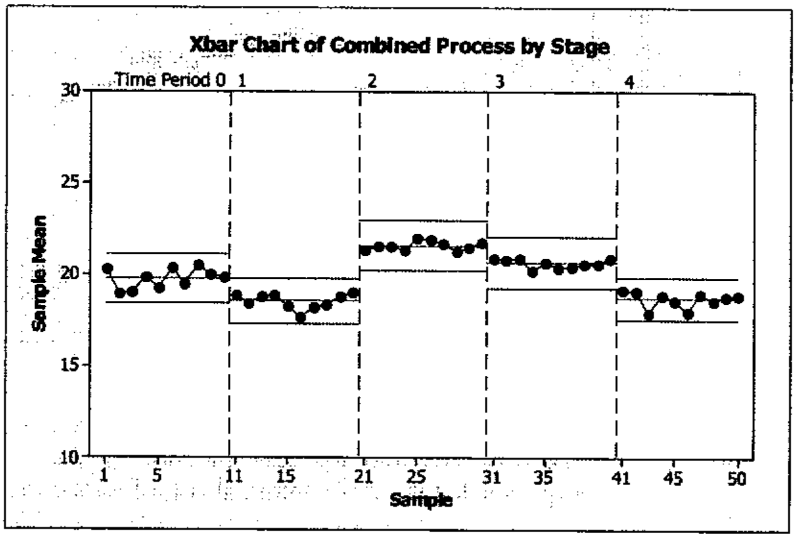


Yet, because the process capability is small when compared to the specifications and the loss function is flat, a histogram of the data suggests that there is a minimal risk of impacting the customer.





The process is being run on one shift per day. When the data are evaluated on that basis, the process exhibited short-term periods of statistical control.



This implies that the process could be monitored using a short run chart (see Chapter III). Other time dependent processes can be monitored by the Individuals and Moving Range Chart, the EWMA Chart, the ARIMA Chart and others.

The charts reveal sensitivity to some special cause. The need for further investigation or process improvement should be considered in the context of business priorities.

The question is whether process parameters are reliable when estimated under such conditions. The answer is no. Deming (1986), Wheeler and Chambers (1992) and Bothe (2002) discuss the risks involved in making capability evaluations when the process lacks statistical control. The consequences of making an erroneous decision based on data from an unstable process can be severe. In general, a stable process is a prerequisite for correctly estimating process capability.

However, in certain situations, such as the time dependent process in the example, the classical indices will provide a conservation estimate of the process performance.

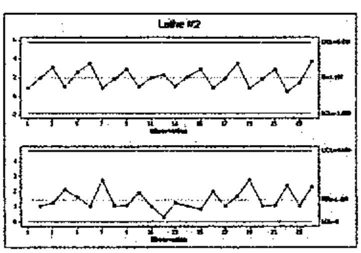
Under certain circumstances, the customer may allow a producer to run a process even though it is a Case 3 process. These circumstances may include:

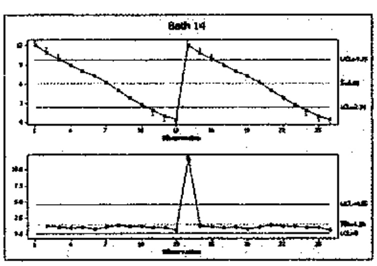
- The customer is insensitive to variation within specifications (see discussion on the loss function in Chapter IV).
- The economics involved in acting upon the special cause exceed the benefit to any and all customers. Economically allowable special causes may include tool wear, tool regrind, cyclical (seasonal) variation, etc.
- The special cause has been identified and has been documented as consistent and predictable.

In these situations, the customer may require the following:

- The process is mature.
- The special cause to be allowed has been shown to act in a consistent manner over a known period of time.
- A process control plan is in effect which will assure conformance to specification of all process output and protection from other special causes or inconsistency in the allowed special cause.

Repeating Patterns





There are times when repetitive patterns are present in control charts due to known assignable causes -- causes that cannot economically be eliminated.

Consider an operation where an outer diameter of a shaft is being machined. As the machining tool wears, the outer diameter will becomes larger. In this example the average chart would have an increasing trend. This trend would continue until the tool is replaced. Over time the average chart will exhibit a sawtooth pattern. As this example highlights, repetitive trends will be present when a process has significant input variables that change consistently overtime which cannot economically be reduced to random causes.

Another example of a process that can produce trends is a process involving chemicals. As parts are processed, the concentration of the chemicals becomes weaker thereby producing a trend. The trend continues until the chemical concentration is brought back to the initial level by process adjustment.

Other examples include processes influenced by ambient temperature, humidity and human fatigue. When these types of repetitive patterns exist, the average chart will exhibit conditions associated with an out-of-control process since there is the (economically influenced) special cause acting on the process. If the influence of this special cause can be shown to be predictable over time and the additional variation is acceptable to the customer, then the process controls can be modified to allow it.

One approach to this is replacing standard control limits with modified control limits. See AT&T (1984), Grant and Leavenworth (1996), Duncan (1986), Charbonneau Webster (1978) for more information on modified control limits.

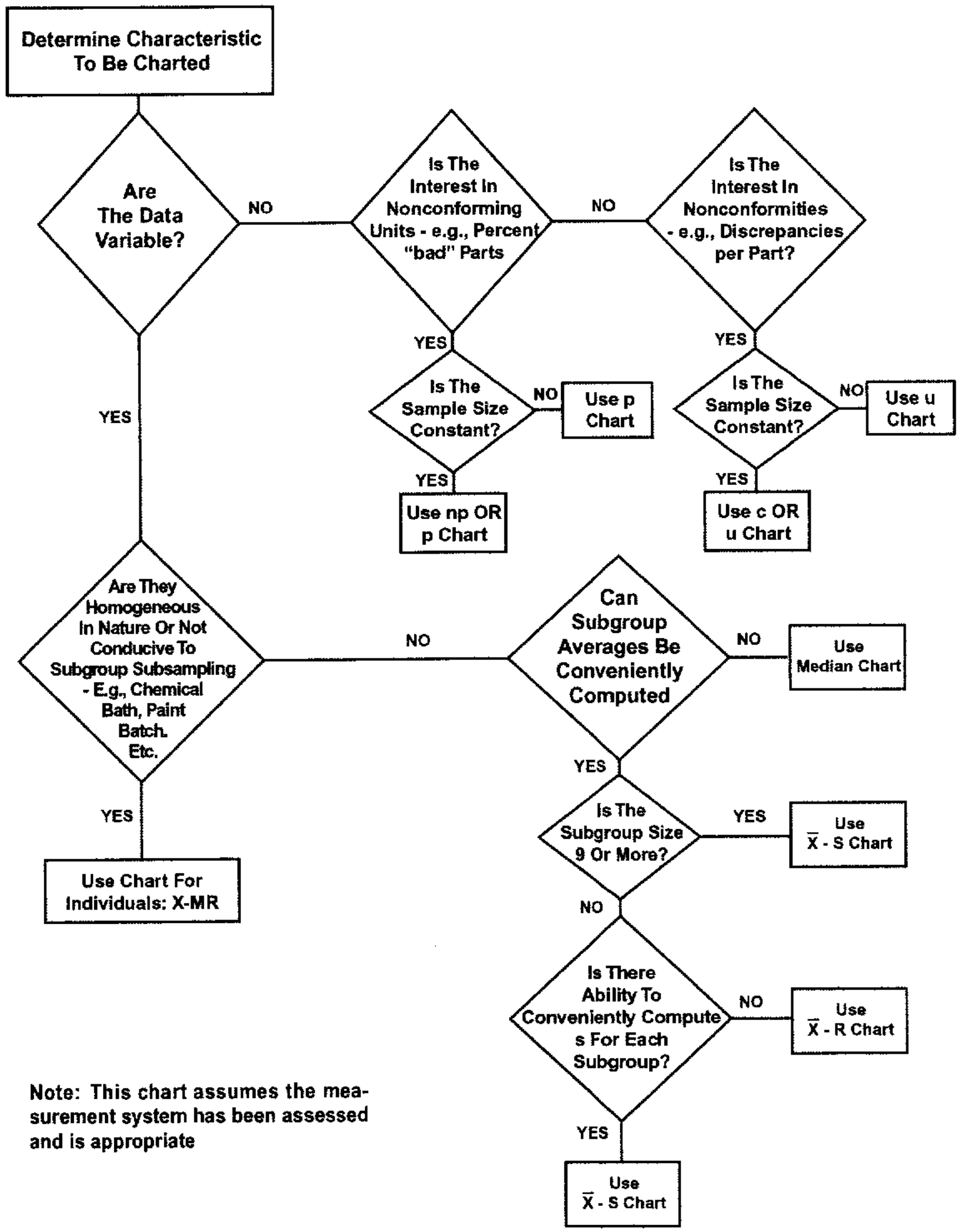
Where modified control limits are used caution should be employed since these charts may fail to disclose the presence or absence of statistical control in the manufacturing process.

An alternate approach is to use the Regression Control Chart discussed in Chapter III.

Besides influencing trends, these types of special causes may also cause a batch to batch mean shift. If this additional variation is acceptable to the customer then the process may be controlled using Short Run Charts discussed in Chapter III.

APPENDIX C

Selection Procedure for the Use of the Control Charts Described in This Manual



Note: This chart assumes the measurement system has been assessed and is appropriate

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APPENDIX D

Relationship Between C_{pm} and Other Indices

The C_{pm} index, often associated with Taguchi's Loss Function, was developed as an alternate way to account for the effect of process centering on estimates of process capability or performance. The C_{pk} and P_{pk} indices focus on the process mean and not the specification target value, while the C_{pm} index focuses on the target value. As discussed in Chapter IV, all four of the standard indices (C_p, C_{pk}, P_p, and P_{pk}) should be evaluated for the same data set to obtain a comprehensive assessment of process capability and performance. A large difference between C_p and C_{pk} or between P_p and P_{pk} is an indication of a centering problem. In contrast, by including the variation between the process mean and the specification target value in the calculation, the C_{pm} index evaluates how well the process meets the specification target whether it is centered or not. See Boyles (1991) and Chan, L. J., S.W. Cheng, and F.A. Spiring (1988) for additional information.

The difference between C_{pm} and other indices discussed in this manual results from the way the standard deviation is calculated. The indices discussed in the text use the standard deviation; i.e., the variation around process mean, $\bar{X}$. C_{pm} uses an analogue based on the target, i.e., the variation around the target, T.

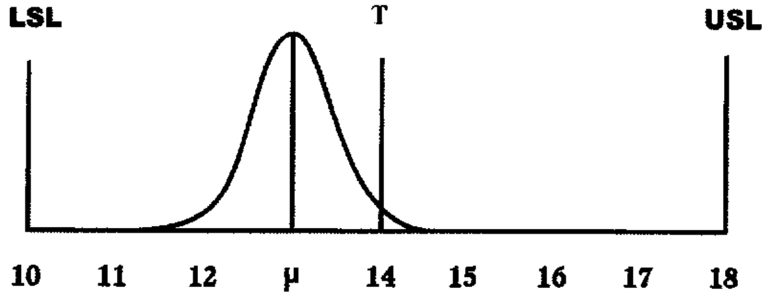
$$C_{pm} = \frac{USL - LSL}{6s_{C_{pm}}}, \quad \text{where} \quad s_{C_{pm}} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - T)^2}{n-1}}$$

The following graphs assume bilateral tolerance:[54] That is,
$(USL - T) = (T - LSL)$

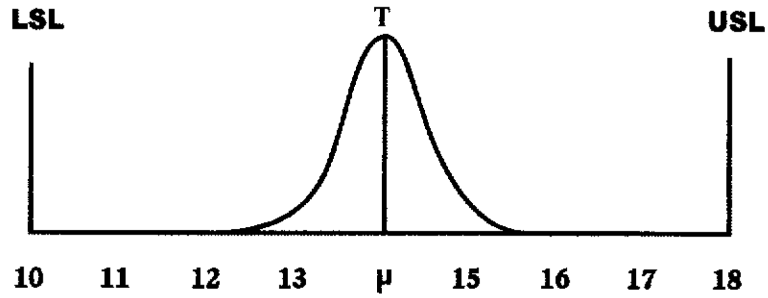
[54] See Bothe (2001) for a discussion of the situation where T is not the middle of the specification.

APPENDIX D

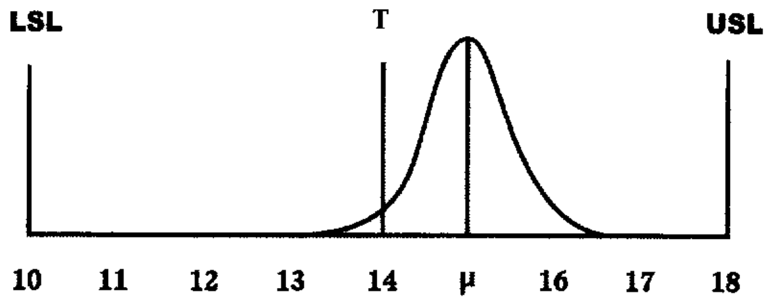
Relationship Between C_{pm} and Other Indices



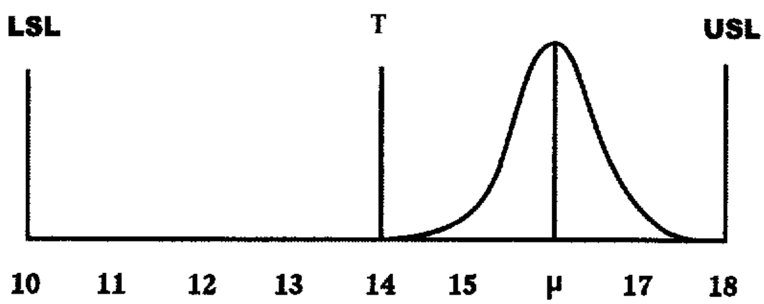
μ	Cp	CPL	CPU	Cpk	Cpm
13	2	1.5	2.5	1.5	1.11



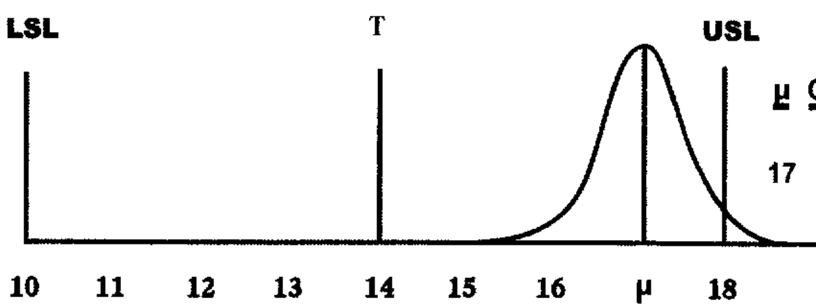
μ	Cp	CPL	CPU	Cpk	Cpm
14	2	2.0	2.0	2.0	2.00



μ	Cp	CPL	CPU	Cpk	Cpm
15	2	2.5	1.5	1.5	1.11



μ	Cp	CPL	CPU	Cpk	Cpm
16	2	3.0	1.0	1.0	0.63



μ	Cp	CPL	CPU	Cpk	Cpm
17	2	3.5	0.5	0.5	0.43

APPENDIX E

Table of Constants and Formulas for Control Charts*

	$\bar{X}$ and R Charts				$\bar{X}$ and s Charts			
	Chart for Averages	Chart for Ranges (R)			Chart for Averages	Chart for Ranges (R)		
	Control Limits Factor	Divisors to Estimate σ_X	Factors for Control Limits		Control Limits Factor	Divisors to Estimate σ_X	Factors for Control Limits	
Subgroup Size	A_2	d_2	D_3	D_4	A_3	c_4	B_3	B_4
2	1.880	1.128	—	3.267	2.659	0.7979	—	3.267
3	1.023	1.693	—	2.574	1.954	0.8862	—	2.568
4	0.729	2.059	—	2.282	1.628	0.9213	—	2.266
5	0.577	2.326	—	2.114	1.427	0.9400	—	2.089
6	0.483	2.534	—	2.004	1.287	0.9515	0.030	1.970
7	0.419	2.704	0.076	1.924	1.182	0.9594	0.118	1.882
8	0.373	2.847	0.136	1.864	1.099	0.9650	0.185	1.815
9	0.337	2.970	0.184	1.816	1.032	0.9693	0.239	1.761
10	0.308	3.078	0.223	1.777	0.975	0.9727	0.284	1.716
11	0.285	3.173	0.256	1.744	0.927	0.9754	0.321	1.679
12	0.266	3.258	0.283	1.717	0.886	0.9776	0.354	1.646
13	0.249	3.336	0.307	1.693	0.850	0.9794	0.382	1.618
14	0.235	3.407	0.328	1.672	0.817	0.9810	0.406	1.594
15	0.223	3.472	0.347	1.653	0.789	0.9823	0,428	1.572
16	0.212	3.532	0.363	1.637	0.763	0.9835	0.448	1.552
17	0.203	3.588	0.378	1.622	0.739	0.9845	0.466	1.534
18	0.194	3.640	0.391	1.608	0.718	0.9854	0.482	1.518
19	0.187	3.689	0.403	1.597	0.698	0.9862	0.497	1.503
20	0.180	3.735	0.415	1.585	0.680	0.9869	0.510	1.490
21	0.173	3.778	0.425	1.575	0.663	0.9876	0.523	1.477
22	0.167	3.819	0.434	1.566	0.647	0.9882	0.534	1.466
23	0.162	3.858	0.443	1.557	0.633	0.9887	0.545	1.455
24	0.157	3.895	0.451	1.548	0.619	0.9892	0.555	1.445
25	0.153	3.931	0.459	1.541	0.606	0.9896	0.565	1.435

	Centerline	Control Limits	
$\bar{X}$ and R Charts	$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$
	$CL_R = \bar{R}$	$UCL_R = D_4\bar{R}$	$LCL_R = D_3\bar{R}$
$\bar{X}$ and s Charts	$CL_{\bar{X}} = \bar{X}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_3\bar{S}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{S}$
	$CL_s = \bar{s}$	$UCL_s = B_4\bar{s}$	$LCL_s = B_3\bar{s}$

* From ASTM publication STP-15D, *Manual on the Presentation of Data and Control Chart Analysis*, 1976; pp 134-136. Copyright ASTM, 1916 Race Street, Philadelphia, Pennsylvania 19103. Reprinted, with permission.

APPENDIX E - Table of Constants and Formulas for Control Charts (Cont.)

	Median Charts**				Charts for Individuals			
	Chart for Medians	Chart for Ranges (R)			Chart for Individuals	Chart for Ranges (R)		
	Control Limits Factor	Divisors to Estimate σ_X	Factors for Control Limits		Control Limits Factor	Divisors to Estimate σ_X	Factors for Control Limits	
Subgroup Size	$\bar{\tilde{A}}_2$	d_2	D_3	D_4	E_2	d_2	D_3	D_4
2	1.880	1.128	—	3.267	2.660	1.128	—	3.267
3	1.187	1.693	—	2.574	1.772	1.693	—	2.574
4	0.796	2.059	—	2.282	1.457	2.059	—	2.282
5	0.691	2.326	—	2.114	1.290	2.326	—	2.114
6	0.548	2.534	—	2.004	1.184	2.534	—	2.004
7	0.508	2.704	0.076	1.924	1.109	2.704	0.076	1.924
8	0.433	2.847	0.136	1.864	1.054	2.847	0.136	1.864
9	0.412	2.970	0.184	1.816	1.010	2.970	0.184	1.816
10	0.362	3.078	0.223	1.777	0.975	3.078	0.223	1.777

	Centerline	**Control Limits**	
Median Charts	$CL_{\tilde{X}} = \bar{\tilde{X}}$	$UCL_{\tilde{X}} = \bar{\tilde{X}} + \bar{\tilde{A}}_2\bar{R}$	$LCL_{\tilde{X}} = \bar{\tilde{X}} - \bar{\tilde{A}}_2\bar{R}$
	$CL_{\bar{R}} = \bar{R}$	$UCL_R = D_4\bar{R}$	$LCL_R = D_3\bar{R}$
Charts for Individuals	$CL_X = \bar{X}$	$UCL_X = \bar{X} + E_2\bar{R}$	$LCL_X = \bar{X} - E_2\bar{R}$
	$CL_R = \bar{R}$	$UCL_R = D_4\bar{R}$	$LCL_R = D_3\bar{R}$

For extended d_2 tables see the MSA Manual 3rd edition.

** $\bar{\tilde{A}}_2$ factors derived from ASTM-STP-15D Data and Efficiency Tables contained in Dixon and Massey (1969), page 488.

APPENDIX E - Table of Constants and Formulas for Control Charts (Cont.)

Attributes Charts

	Centerline	Control Limits	
		Samples not necessarily of constant size	
p chart for proportions of units in a category	$CL_p = \bar{p}$	$UCL_{P_i} = \bar{p} + 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n_i}}$	$LCL_{P_i} = \bar{p} - 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n_i}}$
		If the sample size is constant (n)	
		$UCL_P = \bar{p} + 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$	$LCL_P = \bar{p} - 3\frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$
np chart for number/rate of units in a category	$CL_{np} = \overline{np}$	$UCL_{np} = \overline{np} + 3\sqrt{\overline{np}(1-\frac{\overline{np}}{n})}$ $= \overline{np} + 3\sqrt{\overline{np}(1-\bar{p})}$	$LCL_{np} = \overline{np} - 3\sqrt{\overline{np}(1-\frac{\overline{np}}{n})}$ $= \overline{np} - 3\sqrt{\overline{np}(1-\bar{p})}$
c chart for number of incidences in one or more categories	$CL_C = \bar{c}$	$UCL_C = \bar{c} + 3\sqrt{\bar{c}}$	$LCL_C = \bar{c} - 3\sqrt{\bar{c}}$
u chart for number of incidences per unit in one or more categories	$CL_u = \bar{u}$	Samples not necessarily of constant size	
		$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{n_i}}$ $= \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$	$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{n_i}}$ $= \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$
		Using average sample size	
		$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{\bar{n}}}$ $= \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$	$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{\bar{n}}}$ $= \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$
		If the sample size is constant (n)	
		$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{n}}$ $= \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$	$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{n}}$ $= \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$

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APPENDIX F

Capability Index Calculations Example

For capability indices to be valid, several assumptions should be satisfied (see Chapter IV, Section A and Section B). They are:

- The process from which the data come is statistically stable, that is, the normally accepted SPC rules must not be violated.
- The individual measurements from the process data form an approximately normal distribution.[55]
- A sufficient number of parts must be evaluated in order to capture the variation that is inherent in the process. It is recommended that at least 125 individual values be collected using a subgroup size of five. Other subgroup sizes may be more appropriate for a particular application, but the total sample size should be at least 125.
- The specifications are based on customer requirements.

The following data set is evaluated against these assumptions and, since the assumptions hold, the capability indices are calculated.

[55] For non-normal distributions, see Chapter IV, Section B.

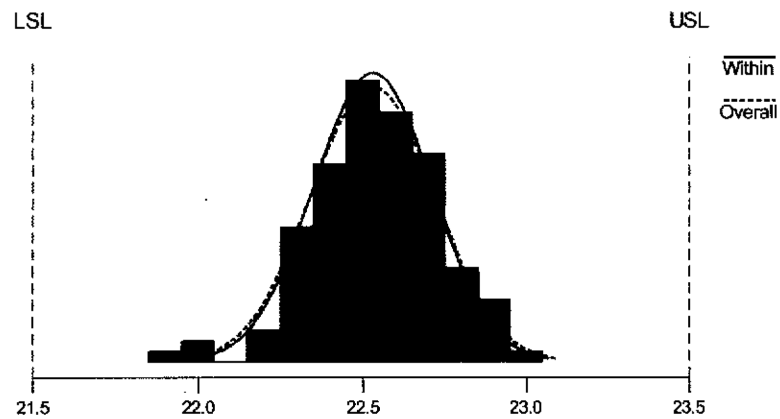
Data Set:

Data Point	Diameter	Data Point	Diameter	Data Point	Diameter	Data Point	Diameter	Data Point	Diameter
1	22.30	26	22.37	51	22.28	76	22.65	101	22.48
2	22.54	27	22.34	52	22.55	77	22.50	102	22.38
3	22.01	28	22.75	53	22.38	78	22.41	103	22.28
4	22.62	29	22.71	54	22.65	79	22.39	104	22.72
5	22.65	30	22.51	55	22.56	80	22.48	105	22.96
6	22.86	31	22.23	56	22.54	81	22.50	106	22.53
7	22.68	32	22.36	57	22.25	82	22.86	107	22.52
8	22.43	33	22.90	58	22.40	83	22.60	108	22.61
9	22.58	34	22.45	59	22.72	84	22.60	109	22.62
10	22.73	35	22.48	60	22.90	85	22.66	110	22.60
11	22.88	36	22.60	61	22.31	86	22.79	111	22.54
12	22.68	37	22.72	62	22.57	87	22.61	112	22.56
13	22.46	38	22.35	63	22.38	88	22.81	113	22.36
14	22.30	39	22.51	64	22.58	89	22.66	114	22.46
15	22.61	40	22.69	65	22.30	90	22.37	115	22.71
16	22.44	41	22.61	66	22.42	91	22.65	116	22.84
17	22.66	42	22.52	67	22.21	92	22.75	117	22.52
18	22.48	43	22.52	68	22.45	93	21.92	118	22.88
19	22.37	44	22.49	69	22.24	94	22.00	119	22.68
20	22.56	45	22.31	70	22.55	95	22.45	120	22.54
21	22.59	46	22.42	71	22.25	96	22.51	121	22.76
22	22.65	47	22.64	72	22.36	97	22.58	122	22.65
23	22.78	48	22.52	73	22.25	98	22.46	123	22.51
24	22.58	49	22.40	74	22.34	99	22.76	124	22.77
25	22.33	50	22.63	75	22.67	100	22.56	125	22.43

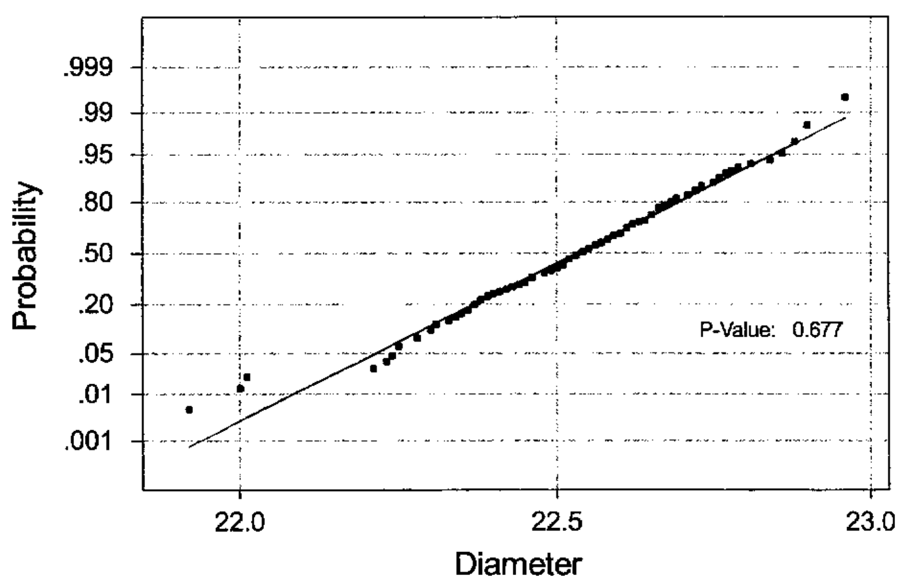
Analysis

Histogram Chart, Normality Plot and the Xbar and R Chart can be used to determine the validity of the first two assumptions.

Histogram of Diameter Data

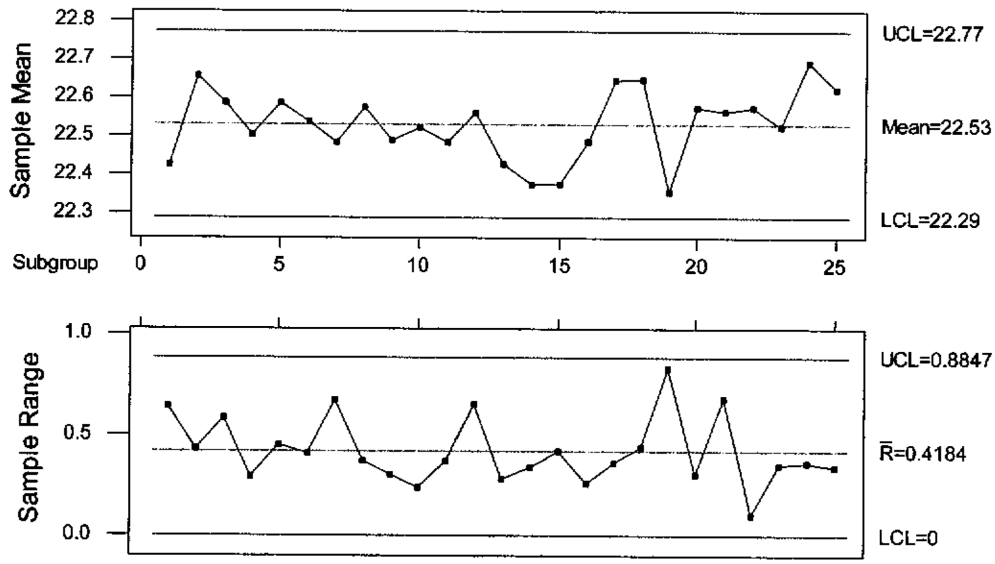


Normal Probability Plot



The above two graphs provide evidence that the data likely came from a normally distributed population.

Xbar/R Chart for Diameter



Control charts provide evidence that the process is in statistical control. Consequently it is appropriate to calculate the indices for this data set.

Diameter Statistics:

Sample size = n =	125
Subgroup Size =	5
Number of Subgroups =	25
Upper Specification Limit =	23.5
Lower Specification Limit =	21.5

The specifications are based on customer and functional requirements.

Within-subgroup standard deviation = $\hat{\sigma}_C = \bar{R}/d_2 = \frac{0.4184}{2.326} = 0.179880$

Total variation standard deviation $= s = \sqrt{\sum_i^n \frac{(x_i - \bar{x})^2}{n-1}} = 0.189037$

The above information is necessary for the evaluation of the indices.

$$C_p = \frac{USL - LSL}{6\left(\bar{R}/d_2\right)} = \frac{23.5 - 21.5}{6 \times 0.179881} = 1.85$$

$$C_{pk} = \text{Minimum}\left(\text{CPL} = \frac{\bar{\bar{X}} - LSL}{3\left(\bar{R}/d_2\right)},\quad \text{CPU} = \frac{USL - \bar{\bar{X}}}{3\left(\bar{R}/d_2\right)}\right)$$

$$= \text{Minimum}\left(\text{CPL} = \frac{22.5308 - 21.5}{3 \times 0.179880},\quad \text{CPU} = \frac{23.5 - 22.5308}{3 \times 0.179880}\right)$$

$$= \text{Minimum}\left(\text{CPL} = 1.91,\quad \text{CPU} = 1.80\right)$$

$$= 1.80$$

$$P_p = \frac{USL - LSL}{6s} = \frac{23.5 - 21.5}{6 \times 0.189037} = 1.76$$

$$P_{pk} = \text{Minimum}\left(\text{PPL} = \frac{\bar{\bar{X}} - LSL}{6s},\quad \text{PPU} = \frac{USL - \bar{\bar{X}}}{6s}\right)$$

$$= \text{Minimum}\left(\text{PPL} = \frac{22.5308 - 21.5}{3 \times 0.189037},\quad PPU = \frac{23.5 - 22.5308}{3 \times 0.189037}\right)$$

$$= \text{Minimum}\left(\text{PPL} = 1.82,\quad \text{PPU} = 1.71\right)$$

$$= 1.71$$

Conclusion:

The following observations are made:

- C_{pk} is approximately equal to C_p, and P_{pk} is approximately equal to P_p. Both of these conditions are indicators that the process is well centered.
- All indices are relatively high indicating that the process is capable of producing near-zero nonconformances if the process remains in statistical control.
- Since the C_p and P_p are approximately equal it implies minimal between-subgroup variation.
- A large discrepancy between C_{pk} and P_{pk} would indicate the presence of excessive between-subgroup variation.
- A large discrepancy between C_p and C_{pk} (or between P_p and P_{pk}) would indicate a process centering problem.

NOTE: The process variability is an integral part of the capability index calculations, hence it is important to be consistent in choosing the method to calculate the within-subgroup variability. As the table below shows there are two ways to estimate the process variability $(\hat{\sigma}_C)$ and its effect on the C_{pk} calculations. Both are correct; i.e., both are valid estimates of the "true" variation. Use $\bar{R}/d_2$ if an $\bar{X}$ *and* R chart is used to collect the data and $\bar{s}/c_4$ if an $\bar{X}$ *and* s chart is used.

Method for calculating $\hat{\sigma}_C$	Summary of Results	
	Within-subgroup Variation	C_{pk}
$\bar{R}/d_2$	0.1799	1.80
$\bar{s}/c_4$	0.1820	1.78

NOTE: The total variation standard deviation value $(\hat{\sigma}_P = 0.1890)$ is not affected by the methodology used to estimate within-subgroup variation.

APPENDIX G

Glossary of Terms and Symbols

Terms Used in This Manual

ARMA Control Chart

The Autoregressive Moving Average Control Chart is a control chart which uses a regression model to account for interrelationship among the data. It may be used in cases where the assumption that the sample data are independent is violated.

Attributes Data

Qualitative data that can be categorized for recording and analysis. Examples include characteristics such as: the presence of a required label, the installation of all required fasteners, the absence of errors on an expense report. Other examples are characteristics that are inherently measurable (i.e., could be treated as variables data), but where the results are recorded in a simple *yes/no* fashion, such as acceptability of a shaft diameter when checked on a go/no-go gage, or the presence of any engineering changes on a drawing. Attributes data are usually gathered in the form of nonconforming units or of nonconformities; they are analyzed by *p, np, c* and *u* control charts (see also Variables Data).

Autocorrelation

The degree of relationship between elements of a stationary time series.

Average (see also Mean)

The sum of values divided by the number (sample size) of values. It is designated by a bar over the symbol for the values being averaged. For example:

- $\bar{X}$ (X-bar) is the average of the X values within a subgroup;
- $\bar{\bar{X}}$ (X double bar) is the average of subgroup averages ($\bar{X}$);
- $\bar{\tilde{X}}$ (X tilde-bar) is the average of subgroup medians;
- $\bar{R}$ (R-bar) is the average of subgroup ranges.

Average Run Length

The number of sample subgroups expected between out-of-control signals. The in-control Average Run Length

	(ARL_0) is the expected number of subgroup samples between false alarms.
Between-subgroup Variation	See **Variation**.
Binomial Distribution	A discrete probability distribution for attributes data that applies to conforming and nonconforming units and underlies the p and np charts.
Cause and Effect Diagram	A simple tool for individual or group problem solving that uses a graphic description of the various process elements to analyze potential sources of process variation. Also called fishbone diagram (after its appearance) or Ishikawa diagram (after its developer).
Centerline	The line on a control chart that represents the average value of the items being plotted.
Characteristic	A distinguishing feature of a process or its output.
Common Cause	A source of variation that affects all the individual values of the process output being studied; this is the source of the inherent process variation.
Confidence Interval	An interval or range of values, calculated from sample data, that contains, with a (100 - α) degree of certainty, the population parameter of interest, e.g., the true population average. α, called the *Level of Significance*, is the probability of committing a Type I error. See Montgomery (1997) or Juran and Godfrey (1999) for calculation methods.
Consecutive	Units of output produced in succession; a basis for selecting subgroup samples.
Continual Improvement	The operational philosophy that makes best use of the talents within the Company to produce products of increasing quality for our customers in an increasingly efficient way that protects the return on investment to our stockholders. This is a dynamic strategy designed to

enhance the strength of the Company in the face of present and future market conditions. It contrasts with any static strategy that accepts (explicitly or implicitly) some particular level of outgoing nonconformances as inevitable.

Control — See **Statistical Control**.

Control Chart — A graphic representation of a characteristic of a process, showing plotted values of some statistic gathered from that characteristic, a centerline, and one or two control limits. It minimizes the net economic loss from Type I and Type II errors. It has two basic uses: as a judgment to determine if a process has been operating in statistical control, and to aid in maintaining statistical control.

Control Limit — A line (or lines) on a control chart used as a basis for judging the stability of a process. Variation beyond a control limit is evidence that special causes are affecting the process. Control limits are calculated from process data and are not to be confused with engineering specifications.

Control Statistic — The statistic used in developing and using a control chart. A value calculated from or based upon sample data (e.g., a subgroup average or range), used to make inferences about the process that produced the output from which the sample came.

Convenience Sampling — A sample scheme wherein the samples are collected using an approach which makes it "easy" to collect the samples but does not reflect the nature of potential special causes which could affect the process. Examples of this are collecting samples just before a break period, or from the top of a bin, pallet or other storage container. This type of sampling is not appropriate for process analysis or control because it can lead to a biased result and consequently a possible erroneous decision.

Correlation — The degree of relationship between variables.

Correlation Matrix — A matrix of all possible correlations of factors under consideration.

CUSUM Control Chart
A control chart approach that uses the current and recent past process data to detect small to moderate shifts in the process average or variability. CUSUM stands for "cumulative sum" of deviations from the target and puts equal weight on the current and recent past data.

Datum
The singular of "data". A single point in a series of data. Not to be confused with the word as used within Geometric Dimensioning & Tolerancing (GD&T).

Detection
A reactive (past-oriented) strategy that attempts to identify unacceptable output after it has been produced and then separate it from acceptable output (see also **Prevention**).

Dispersion
See **Process Spread**.

Distribution
A way of describing the output of a stable system of variation, in which individual values as a group form a pattern that can be described in terms of its location, spread, and shape. Location is commonly expressed by the mean or average, or by the median; spread is expressed in terms of the standard deviation or the range of a sample; shape involves many characteristics such as symmetry (skewness) and peakedness (kurtosis). These are often summarized by using the name of a common distribution such as the normal, binomial, or poisson.

EWMA Control Chart
The Exponentially Weight Moving Average Control Chart is an approach to detect small shifts in the process location. It uses as a statistic to monitor the process location the exponentially weighted moving average.

Haphazard Sampling
A sample scheme wherein the samples are collected using an unsystematic, indiscriminant, unplanned, and/or chaotic approach. This type of sampling is not appropriate for process analysis or control because it can lead to a biased result and consequently a possible erroneous decision.

Individual
A single unit, or a single measurement of a characteristic, often denoted by the symbol X.

Inherent Variation	See **Variation**.
Location	A general term for the typical values of central tendency of a distribution.
Loss Function	A graphical representation of the relationship between the customer's sensitivity (loss) and deviations from the target (design intent). This analysis is conducted without considering the specifications.
Mean	A measure of location. The **average** of values in a group of measurements.
MCUSUM Control Chart	The Multivariate Cumulative Sum Control Chart is the application of the CUSUM Control Chart approach to multivariate situations.
Median	A measure of location. The middle value in a group of measurements, when arranged from lowest to highest. If the number of values is even, by convention the average of the middle two values is used as the median. Subgroup medians form the basis for a simple control chart for process location. Medians are designated by a tilde (~) over the symbol for the individual values: $\tilde{X}$ is the median of a subgroup.
MEWMA Control Chart	The Multivariate Exponentially Weight Moving Average Control Chart is the application of the EWMA Control Chart approach to multivariate situations.
Mode	A measure of location defined by the value that occurs most frequently in a distribution or data set (there may be more than one mode within one data set).
Moving Range	A measure of process spread. The difference between the highest and lowest value among two or more successive samples. As each additional datum point (sample) is obtained, the range associated with that point is computed by adding the new point and deleting the 'oldest' chronological point, so that each range calculation has at least one shared point from the previous range calculation.

Typically, the moving range is used in concert with control charts for individuals and uses two-point (consecutive points) moving ranges.

Multivariate Control Chart

The genre of control charts that have been developed to monitor and control processes that are more appropriately modeled with a multivariate distribution rather than multiple univariate distributions.

Nonconforming Units

Units which do not conform to a specification or other inspection standard; *p* and *np* control charts are used to analyze systems producing nonconforming units.

Nonconformity

A specific occurrence of a condition which does not conform to a specification or other inspection standard. An individual nonconforming unit can have more than one nonconformity. For example, a door could have several dents and dings plus a malfunctioning handle; a functional check of a HVAC unit could reveal any of a number of potential discrepancies. *c* and *u* control charts are used to analyze systems producing nonconformities.

Non-Normal Control Chart

A control chart approach in which adjustments are made to the data or the control limits to allow process control similar to that of Shewhart charts while compensating for the characteristics of a non-normal distribution.

Non-Normal Distribution

A probability distribution that does not follow the normal form; i.e., a distribution where the moments greater than order two are not all zero.

Normal Distribution

A continuous, symmetrical, bell-shaped frequency distribution for variables data that is the basis for the control charts for variables.

Pre-Control

An application of probabilistic analysis to product (nonconformance) control using two data points within each sample.

Operational Definition

A means of clearly communicating quality expectations and performance; it consists of
(1) a criterion to be applied to an object or to a group,
(2) a test of the object or of the group,
(3) a decision: yes or no – the object or the group did or did not meet the criterion.[56]

Over-adjustment

Tampering; taking action on a process when the process is actually in statistical control. Ascribing a variation or a mistake to a special cause, when in fact the cause belongs to the system (common causes).

Pareto Chart

A simple tool for problem solving that involves ranking all potential problem areas or sources of variation according to their contribution to cost or to total variation. Typically, a few causes account for most of the cost (or variation), so problem-solving efforts are best prioritized to concentrate on the "vital few" causes, temporarily ignoring the "trivial many".

Point Estimate

A statistic (single number) calculated from sample data (e.g., average or standard deviation) for which there is some expectation that it is "close" to the population parameter it estimates.

Poisson Distribution

A discrete probability distribution for attributes data that applies to nonconformities and underlies the c and u control charts.

Prediction Interval

Once a regression model is established for a population, the response, y, can be predicted for future values (samples) of the regressor variable(s), $x_0, x_1, \dots x_n$.
The interval for (100 - α) confidence in this prediction is called the prediction interval.

Prevention

A proactive (future-oriented) strategy that improves quality and productivity by directing analysis and action toward correcting the process itself. Prevention is consistent with a philosophy of continual improvement (see also **Detection**).

[56] See Deming (1982).

Probability Based Charts — An approach which uses analysis and charts based on categorical data and the probabilities related to the categories for the control and analysis of products and processes.

Probability Sampling — See **Random Sampling**.

Problem Solving — The process of moving from symptoms to causes (special or common) to actions. Among the basic techniques that can be used are Pareto charts, cause-and-effect diagrams and statistical process control techniques.

Process — The combination of people, equipment, materials, methods, measurement and environment that produce output – a given product or service. A process can involve any aspect of the business. "6M's" is a catch phrase sometimes used to describe a process: Man, Material, Method, Machine, Mother Nature, Measurement.

Process Average — The location of the distribution of measured values of a particular process characteristic, usually designated as an overall average, $\bar{\bar{X}}$.

Process Capability — The $6\hat{\sigma}$ range of inherent process variation.

Variables Data Case
This is defined as $6\hat{\sigma}_C$.

Attributes Data Case
This is usually defined as the average proportion or rate of nonconformances or nonconformities.

Process Control — See **Statistical Process Control**.

Process Performance — The $6\hat{\sigma}$ range of total process variation.

Process Spread — The extent to which the distribution of individual values of the process characteristic vary; often shown as the process average plus or minus some number of standard deviations ($\bar{\bar{X}} \pm 3\hat{\sigma}$).

Quadratic

Of or pertaining to a second order mathematical model; a common graphical example is a parabola.

Randomness

A condition in which no pattern in the data can be discerned.

Random Sampling

A random sample is one in which every sample point has the same chance (probability) of being selected. A random sample is systematic and planned; that is, all sample points are determined before any data are collected.

The process of selecting units for a sample of size *n*, in such a manner that each *n* unit under consideration has an equal chance of being selected in the sample.

Convenience Sampling:

See **Convenience Sampling**

Haphazard Sampling:

See **Haphazard Sampling**

Range

A measure of process spread. The difference between the highest and lowest values in a subgroup, a sample, or a population.

Rational Subgroup

A subgroup gathered in such a manner as to give the maximum chance for the measurements in each subgroup to be alike and the maximum chance for the subgroups to differ one from the other. This subgrouping scheme enables a determination of whether the process variation includes special cause variation.

Regression Control Chart

Regression Control Charts are used to monitor the relationship between two correlated variables in order to determine if and when deviation from the known predictable relationship occurs.

Residuals Control Chart

A chart that monitors a process using the residuals (differences) between a fitted model and the data. A process shift will cause a shift in the mean of the residuals.

Run

A consecutive number of points consistently increasing or decreasing, or above or below the centerline. This can be evidence of the existence of special causes of variation.

Sample

See **Subgroup**.

Shape

A general concept for the overall pattern formed by a distribution of values. Shape involves many characteristics such as symmetry (skewness) and peakedness (kurtosis).

Short Run Control Chart

A control chart approach in which adjustments are made to the data or the control limits to allow process control similar to that of Shewhart charts for processes that only produce a small number of products during a single run

Sigma (σ)

The Greek letter used to designate a standard deviation of a population.

Special Cause

A source of variation that affects only some of the output of the process; it is often intermittent and unpredictable. A special cause is sometimes called assignable cause. It is signaled by one or more points beyond the control limits or a non-random pattern of points within the control limits.

Specification

The engineering requirement for judging acceptability of a particular characteristic. A specification must never be confused with a control limit. Ideally, a specification ties directly to or is compatible with the customer's (internal and/or external) requirements and expectations.

Bilateral:

A bilateral specification identifies requirements at both extremes of the process range. Often referred to as a two-sided specification or tolerance.

Unilateral:

A unilateral specification identifies requirements at only one extreme of the process range. Often referred to as a one-sided specification or tolerance.

Spread — The expected span of values from smallest to largest in a distribution (see also **Process Spread**).

Stability — The absence of special causes of variation; the property of being in statistical control.

Stable Process — A process that is in statistical control.

Standard Deviation — A measure of the spread of the process output or the spread of a sampling statistic from the process (e.g., of subgroup averages).

Statistic — A value calculated from or based upon sample data (e.g., a subgroup average or range) used to make inferences about the process that produced the output.

Statistical Control — The condition describing a process from which the effect of all special causes of variation have been eliminated and only that due to common causes remain; i.e., observed variation can be attributed to a constant system of chance causes. This is evidenced on a control chart by the absence of points beyond the control limits and by the absence of non-random patterns within the control limits.

Statistical Inference — Information about population parameters is estimated or inferred from data obtained from a sample of that population. These inferences can be in the form of a single number *(point estimate)* or a pair of numbers *(interval estimate)*.

Statistical Process Control — The use of statistical techniques such as control charts to analyze a process or its output so as to take appropriate actions to achieve and maintain a state of statistical control and to improve the process capability.

Statistical Tolerance Limits — An interval or range of values that is expected to contain a specified proportion of a population. See Montgomery (1997) or Juran and Godfrey (1999) for calculation methods. See **Tolerance Interval**.

Stoplight Control Chart — A probability based chart approach to process control that uses three categories and double sampling. In this approach the target area is designated green, the warning areas as yellow, and the stop zones as red. The use of these colors gives rise to the "stoplight" designation.

Subgroup — One or more observations or measurements used to analyze the performance of a process. Rational subgroups are usually chosen so that the variation represented within each subgroup is as small as feasible for the process (representing the variation from common causes), and so that any changes in the process performance (i.e., special causes) will appear as differences between subgroups. Rational subgroups are typically made up of consecutive pieces, although random samples are sometimes used.

Tolerance — See **Specification**.

Tolerance Interval — See **Statistical Tolerance Limits**.

Total Process Variation — See **Variation**.

Type I Error — Rejecting an assumption that is true; e.g., taking action appropriate for a special cause when in fact the process has not changed (over-control). This is associated with the producer's or alpha risk.

Type II Error — Failing to reject an assumption that is false; e.g., not taking appropriate action when in fact the process is affected by special causes (under-control). This is associated with the consumer's risk or beta risk.

Unimodal — A distribution is said to be unimodal if it has only one mode.

Variables Data — Quantitative data, where measurements are used for analysis. Examples include the diameter of a bearing journal in millimeters, the closing effort of a door in Newtons, the concentration of electrolyte in percent, or the torque of a fastener in Newton-meters. $\bar{X}$ and R, $\bar{X}$ and s, median and range, and individuals and moving range control charts are used for variables data. See also

Attributes Data. (The term "Variables", although awkward sounding, is used in order to distinguish the difference between something that varies, and the control chart used for data taken from a continuous variable).

Variation

The inevitable differences among individual outputs of a process; the sources of variation can be grouped into two major classes: Common Causes and Special Causes.

Inherent Variation:

That process variation due to common causes only.

Within-subgroup Variation:

This is the variation due only to the variation within the subgroups. If the process is in statistical control this variation is a good estimate of the inherent process variation. It can be estimated from control charts by $\bar{R}/d_2$ or $\bar{s}/c_4$.

Between-subgroup Variation:

This is the variation due to the variation between subgroups. If the process is in statistical control this variation should be zero.

Total Process Variation:

This is the variation due to both within-subgroup and between-subgroup variation. If the process is not in statistical control the total process variation will include the effect of the special cause(s) as well as the common causes. This variation may be estimated by s, the sample standard deviation, using all of the individual readings obtained from either a detailed control chart or a process study: $\hat{\sigma}_P = s = \sqrt{\sum_i^n \frac{(x_i - \bar{x})^2}{n-1}}$ where x_i is an individual reading, $\bar{x}$ is the average of the individual readings, and n is the total number of individual readings.

Within-subgroup Variation

See **Variation**.

Zone Analysis

This is a method of detailed analysis of a Shewhart control chart which divides the $\bar{X}$ chart between the control limits into three equidistant zones above the mean and three equidistant zones below the mean.[57]

[57] See also AT&T. (1984) .

Symbols as Used in This Manual

A_2	A multiplier of $\overline{R}$ used to calculate the control limits for averages; tabled in Appendix E.
$\tilde{\overline{A}}_2$	A multiplier of $\overline{R}$ used to calculate the control limits for medians; tabled in Appendix E.
A_3	A multiplier of s used to calculate the control limits for averages; tabled in Appendix E.
B_3, B_4	Multipliers of $\overline{s}$ used to calculate the lower and upper control limits, respectively, for sample standard deviations; tabled in Appendix E.
c	The number of nonconformities in a sample. The c chart is described in Chapter II, Section C.
$\overline{c}$	The average number of nonconformities in samples of constant size n.
c_4	The divisor of $\overline{s}$ used to estimate the process standard deviation; tabled in Appendix E.
C_p	The capability index for a stable process, typically defined as $\frac{(USL - LSL)}{6\hat{\sigma}_C}$.
C_{pk}	The capability index for a stable process, typically defined as the minimum of CPU or CPL.
CPL	The lower capability index, typically defined as $\frac{(\overline{\overline{X}} - LSL)}{3\hat{\sigma}_C}$.

CPU	The upper capability index, typically defined as $\frac{(USL-\bar{\bar{X}})}{3\hat{\sigma}_C}$.
CR	The capability ratio for a stable process, typically defined as $\frac{1}{C_p}$.
d_2	A divisor of $\bar{R}$ used to estimate the process standard deviation; tabled in Appendix E.
D_3, D_4	Multipliers of $\bar{R}$ used to calculate the lower and upper control limits, respectively, for ranges; tabled in Appendix E.
E_2	A multiplier of the average moving range, $\overline{MR}$,used to calculate control limits for individuals; tabled in Appendix E.
k	The number of subgroups being used to calculate control limits.
LCL	The lower control limit; $LCL_{\bar{X}}$, $LCL_{\bar{R}}$, LCL_p, etc., are, respectively, the lower control limits for averages, ranges, proportion nonconforming, etc.
LSL	The lower engineering specification limit.
MR	The moving range of a series of data points used primarily on a chart for individuals.
n	The number of individuals in a subgroup; the subgroup sample size.

$\overline{n}$	The average subgroup sample size; typically used in attributes charts with varying subgroup sample sizes
np	The number of nonconforming items in a sample of size n. The np chart is described in Chapter II, Section C.
$\overline{np}$	The average number of nonconforming items in samples of constant size n.
p	The proportion of units nonconforming in a sample. The p-chart is discussed in Chapter II, Section C.
$\overline{p}$	The average proportion of units nonconforming in a series of samples.
P_p	The performance index, typically defined as $\frac{(USL-LSL)}{6\hat{\sigma}_P}$.
P_{pk}	The performance index, typically defined as the minimum of PPU or PPL.
PPL	The lower performance index, typically defined as $\frac{(\overline{\overline{X}}-LSL)}{3\hat{\sigma}_P}$.
PPU	The upper performance index, typically defined as $\frac{(USL-\overline{\overline{X}})}{3\hat{\sigma}_P}$.
PR	The performance ratio, typically defined as $\frac{1}{P_p}$.
p_z	The proportion of output beyond a point of interest, such as a particular specification limit, z standard deviation units away from the process average.

R	The subgroup range (highest minus lowest value); the R chart is discussed in Chapter II, Section C.
$\bar{R}$	The average range of a series of subgroups of constant size.
s	The sample standard deviation for subgroups; the s-chart is discussed in Chapter II, Section C. The sample standard deviation for processes; s is discussed in Chapter IV, Section A.
$\bar{s}$	The average sample standard deviation of a series of subgroups, weighted if necessary by sample size.
SL	A unilateral engineering specification limit.
u	The number of nonconformities per unit in a sample which may contain more than one unit. The u chart is discussed in Chapter II, Section C.
$\bar{u}$	The average number of nonconformities per unit in samples not necessarily of the same size.
UCL	The upper control limit; $UCL_{\bar{X}}$, $UCL_{\bar{R}}$, UCL_p, etc., are, respectively, the upper control limits for averages, ranges, proportion nonconforming, etc.
USL	The upper engineering specification limit.
X	An individual value. The chart for individuals is discussed in Chapter II, Section C.
$\bar{X}$	The average of values in a subgroup. The $\bar{X}$-chart is discussed in Chapter II, Section C.

$\overline{\overline{X}}$	The average of subgroup averages (weighted if necessary by sample size); the measured process average.
$\tilde{X}$	The median of values in a subgroup; the chart for medians is discussed in Chapter II, Section C. This is pronounced as "x tilde".
$\bar{\tilde{X}}$	The average of subgroup medians; the estimated process median. This is pronounced as "x tilde bar".
z	The number of standard deviation units from the process average to a value of interest such as an engineering specification. When used in capability assessment, z_{USL} is the distance to the upper specification limit, z_{LSL} is the distance to the lower specification limit, and $z_{\min}$ is the distance to the nearest specification limit.
σ	The Greek letter sigma used to designate a standard deviation of a population.
$\sigma_{\bar{x}}$, σ_R, σ_{np}, etc.	The standard deviation of a statistic based on sample process output, such as the standard deviation of the distribution of subgroup averages the standard deviation of the distribution of subgroup ranges, the standard deviation of the distribution of number of nonconforming items, etc.
$\hat{\sigma}$	An estimate of the standard deviation of a process characteristic.
$\hat{\sigma}_P$	The estimate of the standard deviation of a process using the sample standard deviation of a set of individuals about the average of the set. This is an estimate of the total process variation of the process.
$\hat{\sigma}_C$	The estimate of the standard deviation of a stable process using the average range of subgrouped samples taken from the process, usually within the context of control charts,

where the d_2 factor is tabled in Appendix E. This is the within-subgroup variation and an estimate of the inherent variation of the process.

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APPENDIX H

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[58] See Freund and Williams (1966) for an extensive listing of statistical terms and definitions.
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APPENDIX I

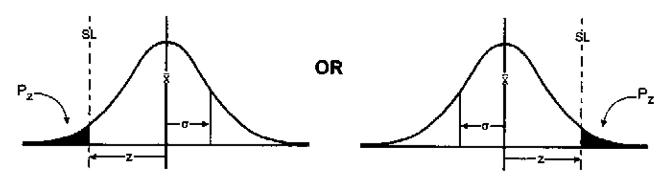
Standard Normal Tables

|z|	x.x0	x.x1	x.x2	x.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
0.0	0.50000000	0.50398940	0.50797830	0.51196650	0.51595340	0.51993880	0.52392220	0.52790320	0.53188140	0.53585640
0.1	0.53982780	0.54379530	0.54775840	0.55171680	0.55567000	0.55961770	0.56355950	0.56749490	0.57142370	0.57534540
0.2	0.57925970	0.58316620	0.58706440	0.59095410	0.59483490	0.59870630	0.60256810	0.60641990	0.61026120	0.61409190
0.3	0.61791140	0.62171950	0.62551580	0.62930000	0.63307170	0.63683070	0.64057640	0.64430880	0.64802730	0.65173170
0.4	0.65542170	0.65909700	0.66275730	0.66640220	0.67003140	0.67364480	0.67724190	0.68082250	0.68438630	0.68793310
0.5	0.69146250	0.69497430	0.69846820	0.70194400	0.70540150	0.70884030	0.71226030	0.71566120	0.71904270	0.72240470
0.6	0.72574690	0.72906910	0.73237110	0.73565270	0.73891370	0.74215390	0.74537310	0.74857110	0.75174780	0.75490290
0.7	0.75803630	0.76114790	0.76423750	0.76730490	0.77035000	0.77337260	0.77637270	0.77935010	0.78230460	0.78523610
0.8	0.78814460	0.79102990	0.79389190	0.79673060	0.79954580	0.80233750	0.80510550	0.80784980	0.81057030	0.81326710
0.9	0.81593990	0.81858870	0.82121360	0.82381450	0.82639120	0.82894390	0.83147240	0.83397680	0.83645690	0.83891290
1.0	0.84134470	0.84375240	0.84613580	0.84849500	0.85083000	0.85314090	0.85542770	0.85769030	0.85992890	0.86214340
1.1	0.86433390	0.86650050	0.86864310	0.87076190	0.87285680	0.87492810	0.87697560	0.87899950	0.88099990	0.88297680
1.2	0.88493030	0.88686060	0.88876760	0.89065140	0.89251230	0.89435020	0.89616530	0.89795770	0.89972740	0.90147470
1.3	0.90319950	0.90490210	0.90658250	0.90824090	0.90987730	0.91149200	0.91308500	0.91465650	0.91620670	0.91773560
1.4	0.91924330	0.92073020	0.92219620	0.92364150	0.92506630	0.92647070	0.92785500	0.92921910	0.93056340	0.93188790
1.5	0.93319280	0.93447830	0.93574450	0.93699160	0.93821980	0.93942920	0.94062010	0.94179240	0.94294660	0.94408260
1.6	0.94520070	0.94630110	0.94738390	0.94844930	0.94949740	0.95052850	0.95154280	0.95254030	0.95352130	0.95448600
1.7	0.95543450	0.95636710	0.95728380	0.95818490	0.95907050	0.95994080	0.96079610	0.96163640	0.96246200	0.96327300
1.8	0.96406970	0.96485210	0.96562050	0.96637500	0.96711590	0.96784320	0.96855720	0.96925810	0.96994600	0.97062100
1.9	0.97128340	0.97193340	0.97257110	0.97319660	0.97381020	0.97441190	0.97500210	0.97558080	0.97614820	0.97670450
2.0	0.97724990	0.97778440	0.97830830	0.97882170	0.97932480	0.97981780	0.98030070	0.98077380	0.98123720	0.98169110
2.1	0.98213560	0.98257080	0.98299700	0.98341420	0.98382260	0.98422240	0.98461370	0.98499660	0.98537130	0.98573790
2.2	0.98609660	0.98644740	0.98679060	0.98712630	0.98745450	0.98777550	0.98808940	0.98839620	0.98869620	0.98898930
2.3	0.98927590	0.98955590	0.98982960	0.99009690	0.99035810	0.99061330	0.99086250	0.99110600	0.99134370	0.99157580
2.4	0.99180250	0.99202370	0.99223970	0.99245060	0.99265640	0.99285720	0.99305310	0.99324430	0.99343090	0.99361280
2.5	0.99379030	0.99396340	0.99413230	0.99429690	0.99445740	0.99461390	0.99476640	0.99491510	0.99506000	0.99520120
2.6	0.99533880	0.99547290	0.99560350	0.99573080	0.99585470	0.99597540	0.99609300	0.99620740	0.99631890	0.99642740
2.7	0.99653300	0.99663580	0.99673590	0.99683330	0.99692800	0.99702020	0.99710990	0.99719720	0.99728210	0.99736460
2.8	0.99744490	0.99752290	0.99759880	0.99767260	0.99774430	0.99781400	0.99788180	0.99794760	0.99801160	0.99807380
2.9	0.99813420	0.99819290	0.99824980	0.99830520	0.99835890	0.99841110	0.99846180	0.99851100	0.99855880	0.99860510
3.0	0.99865010	0.99869380	0.99873610	0.99877720	0.99881710	0.99885580	0.99889330	0.99892970	0.99896500	0.99899920

APPENDIX I
Standard Normal Tables

|z|	x.x0	x.x1	x.x2	x.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
3.1	0.99903240	0.99906460	0.99909570	0.99912600	0.99915530	0.99918360	0.99921120	0.99923780	0.99926360	0.99928860
3.2	0.99931290	0.99933630	0.99935900	0.99938100	0.99940240	0.99942300	0.99944290	0.99946230	0.99948100	0.99949910
3.3	0.99951660	0.99953350	0.99954990	0.99956580	0.99958110	0.99959590	0.99961030	0.99962420	0.99963760	0.99965050
3.4	0.99966310	0.99967520	0.99968690	0.99969820	0.99970910	0.99971970	0.99972990	0.99973980	0.99974930	0.99975850
3.5	0.99976740	0.99977590	0.99978420	0.99979220	0.99979990	0.99980740	0.99981460	0.99982150	0.99982820	0.99983470
3.6	0.99984090	0.99984690	0.99985270	0.99985830	0.99986370	0.99986890	0.99987390	0.99987870	0.99988340	0.99988790
3.7	0.99989220	0.99989640	0.99990040	0.99990430	0.99990800	0.99991160	0.99991500	0.99991840	0.99992160	0.99992470
3.8	0.99992770	0.99993050	0.99993330	0.99993590	0.99993850	0.99994090	0.99994330	0.99994560	0.99994780	0.99994990
3.9	0.99995190	0.99995390	0.99995570	0.99995750	0.99995930	0.99996090	0.99996250	0.99996410	0.99996550	0.99996700
4.0	0.99996830	0.99996960	0.99997090	0.99997210	0.99997330	0.99997440	0.99997550	0.99997650	0.99997750	0.99997840
4.1	0.99997930	0.99998020	0.99998110	0.99998190	0.99998260	0.99998340	0.99998410	0.99998480	0.99998540	0.99998610
4.2	0.99998670	0.99998720	0.99998780	0.99998830	0.99998880	0.99998930	0.99998980	0.99999020	0.99999070	0.99999110
4.3	0.99999150	0.99999180	0.99999220	0.99999250	0.99999290	0.99999320	0.99999350	0.99999380	0.99999410	0.99999430
4.4	0.99999460	0.99999480	0.99999510	0.99999530	0.99999550	0.99999570	0.99999590	0.99999610	0.99999630	0.99999640
4.5	0.9999966023	0.9999967586	0.9999969080	0.9999970508	0.9999971873	0.9999973177	0.9999974423	0.9999975614	0.9999976751	0.9999977838
4.6	0.9999978875	0.9999979867	0.9999980813	0.9999981717	0.9999982580	0.9999983403	0.9999984190	0.9999984940	0.9999985656	0.9999986340
4.7	0.9999986992	0.9999987614	0.9999988208	0.9999988774	0.9999989314	0.9999989829	0.9999990320	0.9999990789	0.9999991235	0.9999991661
4.8	0.9999992067	0.9999992453	0.9999992822	0.9999993173	0.9999993508	0.9999993827	0.9999994131	0.9999994420	0.9999994696	0.9999994958
4.9	0.9999995208	0.9999995446	0.9999995673	0.9999995889	0.9999996094	0.9999996289	0.9999996475	0.9999996652	0.9999996821	0.9999996981
5.0	0.9999997133	0.9999997278	0.9999997416	0.9999997548	0.9999997672	0.9999997791	0.9999997904	0.9999998011	0.9999998113	0.9999998210
5.1	0.9999998302	0.9999998389	0.9999998472	0.9999998551	0.9999998626	0.9999998698	0.9999998765	0.9999998830	0.9999998891	0.9999998949
5.2	0.9999999004	0.9999999056	0.9999999105	0.9999999152	0.9999999197	0.9999999240	0.9999999280	0.9999999318	0.9999999354	0.9999999388
5.3	0.9999999421	0.9999999452	0.9999999481	0.9999999509	0.9999999535	0.9999999560	0.9999999584	0.9999999606	0.9999999628	0.9999999648
5.4	0.9999999667	0.9999999685	0.9999999702	0.9999999718	0.9999999734	0.9999999748	0.9999999762	0.9999999775	0.9999999787	0.9999999799
5.5	0.9999999810	0.9999999821	0.9999999831	0.9999999840	0.9999999849	0.9999999857	0.9999999865	0.9999999873	0.9999999880	0.9999999886
5.6	0.9999999893	0.9999999899	0.9999999905	0.9999999910	0.9999999915	0.9999999920	0.9999999924	0.9999999929	0.9999999933	0.9999999936
5.7	0.9999999940	0.9999999944	0.9999999947	0.9999999950	0.9999999953	0.9999999955	0.9999999958	0.9999999960	0.9999999963	0.9999999965
5.8	0.9999999967	0.9999999969	0.9999999971	0.9999999972	0.9999999974	0.9999999975	0.9999999977	0.9999999978	0.9999999979	0.9999999981
5.9	0.9999999982	0.9999999983	0.9999999984	0.9999999985	0.9999999986	0.9999999987	0.9999999987	0.9999999988	0.9999999989	0.9999999990
6.0	0.9999999990									

The tabled values are 1 - P_Z = the proportion of process output beyond a particular value of interest (such as a specification limit) that is z standard deviation units away from the process average (for a process that is in statistical control and is normally distributed). For example, if z = -2.17, P_Z = 1 - 0.98499660 = 0.0150 or 1.5%. In any actual situation, this proportion is only approximate.



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S.P.C. Manual User Feedback Process

Consistent with the concept of continual improvement, this automotive industry Statistical Process Control (SPC) manual is being subjected to a formal periodic review/revision process. In line with the concept of customer satisfaction, this review will entail consideration of not only any applicable vehicle manufacturer requirement changes from year to year but also of feedback from users of the manual for the purpose of making it more value-added and effective to the automotive industry and user communities. Accordingly, please feel free to offer, in writing, your feedback comments, both pro and con, relative to the manual's understandability, "user-friendliness," etc., in the area indicated below. Please indicate specific manual page numbers where appropriate. Forward your feedback to the address indicated below:

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